A class of nonfactorizable corrections to the pion form factor

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It follows from the particular analytic structure of the Feynman diagrams that the gluon corrections to the asymptotic behavior of the pion form factor vanish in the Glauber region.

The asymptotic behavior of the electromagnetic form factor of the pion,[1-3] $F(Q^2)$, is calculated by means of the operator expansion

$$(p+p')_\nu F(Q^2) \to \sum_{n,m} T^\nu_{m,n}(Q^2)\tilde{c}^*_m c_n, \quad Q^2 \to \infty \tag{1}$$

(see also Fig. 1), where $c_n = \langle 0|\hat{O}_n|\pi(p)\rangle$ are the matrix elements of the local opera-

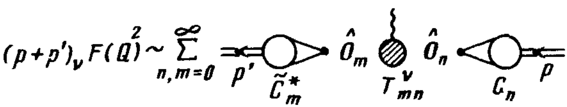

FIG. 1.

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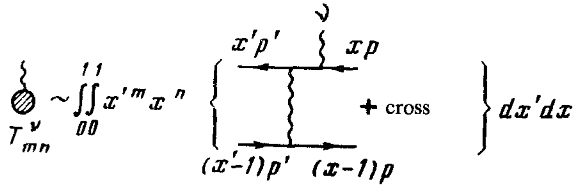

FIG. 2.

tors of twist 2, which are related to the axial current; $T^{\nu}_{m,n}$ is the coefficient function (the "hard block"), which is determined by the moments of the quark-gluon diagram in Fig. 2 in the fraction of the momentum x (x') of the quark in the initial (final) state; and $Q^2 = -(p'-p)^2 > 0$ [see Refs. 4 and 5 for more details on the operator expansion in the form in (1)]. A necessary condition for the use of the operator expansion is that there must be no nonfactorizable corrections of the type in Fig. 3. We can state reasons why the class of corrections of this type is actually not important.

We can ignore the mass of the pion and assume $p = (p_-, 0, \vec{0}_\perp), p' = (0, p'_+, \vec{0}_\perp)$ ($p_- = p'_+ = Q/\sqrt{2}$). We use the Feynman gauge. In Fig. 3 the gluon must be longitudinal (the density matrix of the gluon and the polarization indices corresponding to the left and right blocks are shown in this figure). A large kinematic factor $Q^2 \sim p_- p'_+$ arises in this case (the factor p_- comes from the right block, while p'_+ comes from the left block). We assume

$$q = \alpha p + \beta p' + q_\perp, \qquad k_j = x_j p + y_j p' + k_{\perp j}\,. \tag{2}$$

Over most of the integration range in the operator expansion we have $x_1 = 1$, $y_1 \sim k^2_{\perp 1/Q^2} \ll 1$; $y_2 \sim 1, x_2 \sim (k^2_{\perp 2/Q^2}) \ll 1$. In the integration over the gluon we have $q_\perp^2 \ll Q^2$. For α and β the following possibilities exist: 1) $\alpha \sim 1$, $\beta \sim (q_\perp^2/Q^2)$ or $\beta \sim 1$, $\alpha \sim (q_\perp^2/Q^2)$. This region gives a factorizable contribution ("collinear" logarithms), which is embodied in the operator expansion. 2) $\alpha \ll 1, \beta \ll 1$ with $Q^2\alpha\beta = -q_\perp^2$. This is a doubly logarithmic region. The contribution from it apparently cancels out because the pion is colorless (see Refs. 1–4; the cancellation has been checked up to two-loop corrections inclusively). There are no other sources of logarithmic corrections from the gluon q.

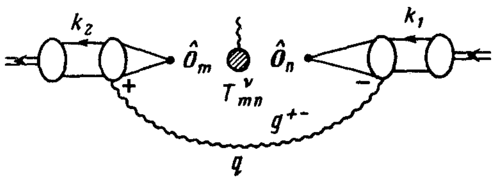

FIG. 3.

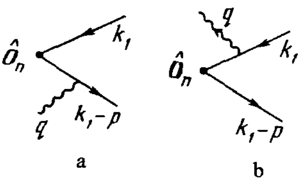

FIG. 4.

In principle, however, there is the possiblity of obtaining large nonfactorizable corrections in the case in which the integral over q is not logarithmic and is instead concentrated in the region $q_\perp^2 \sim m^2$ ($m \sim 1$ GeV will be used below as a typical hadronic value). In this case we have (see the discussion below) $\alpha \sim \beta \sim (m^2/Q^2)$, i.e., $q^2 = q_\perp^2$. We call this region the "Glauber region" (cf. Refs. 6 and 7). In it the integral over q corresponding to Fig. 3 is proportional to

$$I \sim \int_{-\infty}^{\infty} Q^2 d\alpha\, L\,(Q^2\alpha, q_\perp, k_{\perp i} \ldots \quad \frac{d^2 q_\perp}{q_\perp^2} \int_{-\infty}^{\infty} Q^2 d\beta\, R\,(Q^2\beta, q_\perp, k_{\perp j} \ldots), \tag{3}$$

where the functions L and R are related to the left and right blocks in Fig. 3. One factor of Q^2 in (3) stems from the phase volume $(d^4q = \frac{Q^2}{2} d\alpha d\beta d^2 q_\perp)$, while the other is a kinematic factor which results from the longitudinal nature of the gluon. The integral I converges as $q_\perp \to 0$, since we have $R \sim (q_\perp / k_\perp)$ in the limit $q_\perp \to 0$, because of the gauge invariance (the same is true for L). For $q_\perp \sim k_{\perp j} \sim m$ and $Q^2\alpha \sim Q^2\beta \sim m^2$ the integral I contains no parametric small factor of any sort.

We will show below, however, that the integrals over α and β vanish because of analytic considerations. For definiteness, we consider the integral over β. In the emission of a gluon directly from a quark line touching the hard block (Fig. 4) a contribution from the separate diagram does exist (it is determined by the semi-residue at the corresponding pole), but the contributions of Figs. 4a and 4b cancel out. This fact is interpreted in the following way: A pair of quarks produced in a hard block does not interact in the Glauber region over times $\sim 1/Q$ because the system is small and colorless.

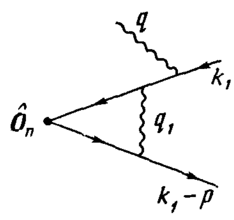

FIG. 5.

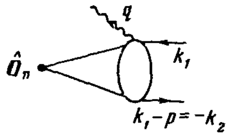

FIG. 6.

If the quarks exchange a gluon q_1 with $q_{\perp 1} \sim m$ before the emission of the gluon q, the integral over β_1 will be nonzero only if $(x_1 - 1) < \alpha_1 < x_1$ (Fig. 5). Under this condition, however, all the poles in β lie on the same side of the real axis in the complex β plane, and the integral over β vanishes. This fact is interpreted as follows. Even before the hadronization stage, one pair of quarks increases in size and acquires the necessary multipole moments, but over this time the two pairs, which correspond to the initial and final pions, respectively, manage to move apart and cannot interact.

This vanishing of the integral over β is a general effect. The integration of the exact amplitude R (Fig. 6) occurs at fixed values of $s = (k_1 + k_2)^2$ and $q^2 \approx q_\perp^2 < 0$. There are, however, changes in the virtuality $(p - q)^2 = q_\perp^2 - Q^2_\beta$ and in $t = (q - k_1)^2 \sim q_\perp^2 - Q^2 x_1 \beta$ and $u(q - k_2)^2 \sim q_\perp^2 - Q^2 x_2 \beta$. Since at $t < 0$, u and $(p - q)^2$ are also negative it can be expected that there will be no left-hand cut in the complex t plane, so that the contour can be closed at infinity, and the integral will therefore vanish. Since this vanishing is a consequence of the analytic structure, it is sufficient to replace the calculation of the amplitude in Fig. 6 by the establishment of this fact for the diagram in Fig. 7, which has all types of singularities that are found in the exact amplitude. We have verified that this vanishing does in fact occur for the diagram in Fig. 7, as it does for all other diagrams of the single- and two-loop approximations. (The analysis reduces to identifying the positions of the poles in β, β_1, and β_2 and is similar to the analysis of the simple case corresponding to Fig. 5.) Since only the analytic properties are important to the analysis, we can assert that the vanishing of integral (3) is a consequence of causality.

Ryskin and Dokshitser[8] have pointed out a phenomenon analogous to that discussed here in connection with the inclusive production of a hadron in e^+e^- annihilation.

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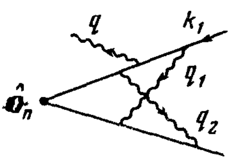

FIG. 7.

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