Cyclotron parametric resonance in "dirty" semiconductors

I. E. Aronov and É. A. Kaner
Institute of Radio Physics and Electronics, Academy of Sciences of the Ukrainian SSR

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The existence of a cyclotron parametric resonance (CPR) in semiconductors with a short mean-free path is predicted. In contrast to the classical parametric instability in mechanics, the conditions of resonant excitation of the electronic subsystem are much less stringent and fully attainable. The width of the instability zone and the shape of the absorption curve are determined.

It has been established elsewhere that the width of the instability zone of the main parametric resonance in mechanical systems is determined by the friction coefficient. At a cyclotron parametric resonance (CPR) in semiconductors, the relaxation rate of the electron momentum ν_i plays the role of friction.[1] In this letter we show that in contrast to the parametric resonance in mechanics, the cyclotron parametric instability can in fact occur in a "dirty" semiconductor with a large value of ν_i, when the condition

$$\Omega_1 \ll \nu_i \ll \Omega_0 , \tag{1}$$

instead of the usual requirement $\Omega_1 > 2\nu_i$, is satisfied. Here Ω_1 is the modulation amplitude of the collective cyclotron rotation in an alternating magnetic field, $\Omega(t) = \Omega_0 + \Omega_1 \cos \gamma t$, γ is the modulation frequency, and Ω_0 is the cyclotron frequency for the constant component of the magnetic field. Cyclotron parametric resonance occurs in the electronic system if $\Omega_0 \cong \gamma/2$.

Remarkably, in spite of the random motion of electrons on the slow time scale, when inequalities (1) are satisfied, the CPR nevertheless occurs if

$$\Omega_1 > (\nu_e \nu_i)^{1/2} . \tag{2}$$

This requirement is easily satisfied together with (1), even in dirty semiconductors, because of the quasielastic nature of the scattering of electrons, since the rate of energy relaxation ν_e is generally much lower than ν_i. Under conditions (1) and (2) the presence of CPR is evident from the nonequilibrium state and anisotropy of the electron distribution.

We will study the influence of strong scattering of electrons on the CPR by using the collision integral $\hat{I}\{f\}$ in the form

$$\hat{I}\{f\} = -\nu_e [\psi_+(\mathbf{p}) - f_0(p)] - \nu_{i1} [\psi_+(\mathbf{p}) - \langle \psi_+(\mathbf{p}) \rangle] - \nu_{i2}\, \psi_-(\mathbf{p}) . \tag{3}$$

Here $\psi_\pm(\mathbf{p}) = (1/2)[f(t,\mathbf{r},\mathbf{p}) \pm f(t,\mathbf{r},-\mathbf{p})]$, $f_0(p)$ is the equilibrium distribution function, and $\langle \psi_+(\mathbf{p}) \rangle$ is the symmetrical part of the distribution function averaged over the angular variables in $\mathbf{p}$ space. The first term on the right side of (3), which is proportional to ν_e, describes the energy relaxation, while the second and third terms describe the momentum relaxation of the symmetrical and asymmetrical parts of the electron distribution function, respectively. The quantity ν_{i_1} is expressed in terms of the total scattering cross section, whereas ν_{i_2} is expressed in terms of the transport cross section. The momentum relaxation rates ν_{i_1} and ν_{i_2} have the same order of magnitude and the same energy dependence for the same scattering mechanisms; they can differ from each other by only a factor on the order of unity.

Collision integral (3) is simpler than the exact expression. It can be shown, however, that the macroscopic electronic characteristics (for example, the average energy, current density, etc.) found by making use of Eq. (3), coincide with the analogous quantities obtained from the exact collision integral.

The use of the kinetic equation with the collision integral (3) makes it possible to analyze explicitly the effect of collisions between electrons and volume scatterers on the CPR. This analysis shows that the parametric resonance indeed exists if inequalities (1) and (2) hold.

We present the results for CPR in a two-dimensional electronic system. The electrons can be assumed to be two-dimensional if the thickness of the plate, d, along the magnetic-field vector is smaller than the cooling length $L = v(\nu_e \nu_i)^{-1/2}$ (v is the average thermal velocity). The width of the instability zone for the main parametric resonance ($\Omega_0 \cong \gamma/2$) is determined

$$\Omega_1^2 - (\nu_e / \nu_i)(2\Omega_0 - \gamma)^2 > \nu_e \nu_i . \tag{4}$$

We wish to emphasize the fundamental difference between this result and the condition of resonant excitation of the electronic system in the quasiballistic state with $\Omega_1 > 2\nu_i$.[1] It is evident from Ref. 4 that the CPR can also be observed in "dirty" semiconductors (semiconductors with a short mean-free path) when the electromagnetic pumping parameter Ω_1 is much lower than the momentum relaxation rate ν_i (1). The conditions of resonant excitation are thus much less stringent than those obtained in Ref. 1.

It is also evident from (4) that quasielastic scattering ($\nu_e \ll \nu_i$) widens the CPR zone, because the strong elastic scattering causes electrons repeatedly to return to the region of the phase space where resonant acceleration of the particles occurs; i.e., the collisions that cause a large number of carriers to become resonant.

In the "dirty" limit (1) the CPR is accompanied by a sharp increase of the average energy of electrons, i.e., strong overheating of the electronic subsystem should be observed at resonance. The CPR can be observed experimentally by measuring the time-averaged specific rf power $W(\Delta)$, absorbed by the semiconductor, as a function of the loss of resonance

$$W(\Delta) = NT\nu_e a^{-1} \rho(\Delta) [(1+\Delta^2)^{-1} + a (1 - \rho^{-1})];$$
$$\Delta = (\Omega_0 - \gamma/2)/\nu_i, \quad a = (\nu_e \nu_i)/\Omega_1^2. \tag{5}$$

where N is the electron density; T is the electron temperature; a is the supercriticality parameter (2), which determines the CPR intensity; the function $\rho(\Delta)$ describes the shape of the resonance line:

$$\rho(\Delta) = \frac{\kappa^2 - 1}{\kappa} \left[\int_0^1 \frac{d\xi\, \xi^{(\kappa-1)/2}}{(\xi + 1/z)^2} + \frac{2}{\kappa + 3} \right];$$

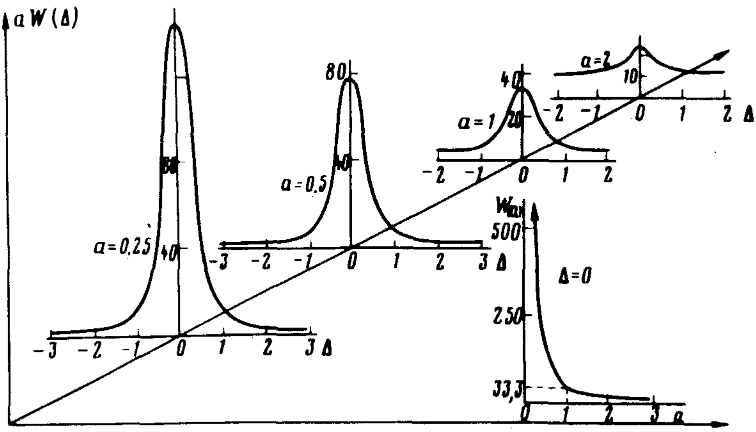

FIG. 1. The specific absorbed power as a function of the constant magnetic field for different pumping intensities. The inset: The amplitude of the resonance as a function of the parameter $a = (\nu_e \nu_i)/\Omega_1^2$.

$$z = \frac{\Omega_1 \epsilon_g}{8\Omega_0 T}\left(\frac{\nu_i}{2\nu_e}\right)^{1/2} \gg 1; \tag{6}$$

$$\kappa = [1 + 8a(1 + 4\Delta^2)]^{1/2}, \qquad \kappa - 1 > (2\ln z)^{-1};$$

and ϵ_g is the width of the forbidden band. The large parameter z is the ratio of the characteristic energy of electrons ϵ_* at CPR to the temperature T. The quantity ϵ_* represents the energy of the limit cycle, which is "smeared" as a result of frequent collisions between electrons. This cycle appears as a result of nonlinear dynamics of the resonant particles stemming from the deviation of their dispersion law from the parabolic law.

At resonance the absorption exhibits a sharp peak at $\Delta = 0$. The width of the resonance curve as a function of Δ is determined primarily by the momentum relaxation rate ν_i. The amplitude of the resonance decreases sharply as the parameter a increases and passes through the value $a = 1$. Figure 1 shows $W(\Delta)$ curves for different values of a; the inset shows how the amplitude of the resonance $W(0)$ changes as a is increased. It is evident that the resonance is clearly observed even at the boundary of the instability zone ($a = 1$). The amplitude of this resonance increases sharply [in proportion to $z^{(3-\sqrt{1+8a})/2}$] with increasing pumping intensity Ω_1^2.

The predicted effect can be observed in relatively impure samples (for example, InSb) at pumping levels of $\Omega_1 \propto H_1 \cong 0.5$–2 Oe and frequencies of $\gamma/2 \cong \Omega_0 \cong 10^{12}$ rad/s in a constant magnetic field of several kilo-oersteds.

[1]I. E. Aronov, E. A. Kaner, and A. A. Slutskin, Solid State Commun. 38, 245 (1981).

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