Long-lived induction signal and spatially nonuniform spin precession in ^{3}He-B

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An explanation is proposed for the anomalously long existence of an induction signal, which has been observed in pulsed NMR experiments in the B phase of ^{3}He, and for the recently discovered time evolution of the frequency of this signal. The explanation is based on an analysis of the spatial nonuniformity of the steady-state distribution of the precessing spins.

The long-lived induction signal is an effect observed in a pulsed NMR study of the superfluid B phase of ^{3}He (Refs. 1 and 2): When the spin is deflected a large angle ($\sim 90°$) from its equilibrium direction, the induction signal is observed for a time far longer than would be expected on the basis of the existing nonuniformity of the magnetic field $\mathbf{H}_0$. Simple estimates show that the spatial "stiffness" of the condensate is one or two orders of magnitude too weak to prevent a loss of phase coherence of the magnetization precession and the disappearance of the induction signal for the magnetic field nonuniformities which are commonly encountered in experiments.

Borovik-Romanov *et al.*[3] recently carried out a systematic study of the long-lived induction signal and obtained results that explain the effect. The observations in Ref. 3 of most importance for the explanation proposed here are as follows:

1. The long-lived induction signal is found at magnetic field gradients up to ~ 1 Oe/cm.

2. In the presence of a magnetic field gradient, the frequency of the long-lived induction signal changes over time, at a rate which depends on the magnitude of the gradient.

The first of these observations identifies the problem which must be solved for a theoretical interpretation of the experiments. Specifically, we must solve Leggett's equations[4] describing the steady-state precession of spins in a spatially nonuniform magnetic field. For a subsequent quantitative comparison with the experiments of Ref. 3, we assume that the Larmor frequency ω_L varies linearly with the coordinate along the magnetic field direction (the z axis); i.e., we assume $\omega_L = \omega_L^0 + z\nabla\omega_L$. It can be shown[5] that the solution we seek must be the minimum of the total free energy of the ^{3}He in the volume of the test chamber

$$F = \int \left\{ \frac{g^2 S^2}{2\chi} - g\mathbf{S}\cdot\mathbf{H} + U + G - \Delta\cdot S_z \right\} dV. \tag{1}$$

The first four terms in braces (curly brackets) constitute Leggett's Hamiltonian in the usual notation with a gradient energy G; the last term is added to reflect conservation of the total longitudinal component of the spin; Δ is a Lagrange multiplier. We introduce the spherical coordinates of the spin: $S = |\mathbf{S}|$, α (the phase of the precession), and β (the angle between $\mathbf{S}$ and the z axis). To describe the motion of the order parameter we must introduce yet another angle: ϕ, the relative phase of the precession of $\mathbf{S}$ and of the rotation of the order parameter around $\mathbf{S}$ (Ref. 5). The dipole energy U in the B phase depends on only β and ϕ:

$$U \sim \left[\cos\beta - \frac{1}{2} + (1+\cos\beta)\cos\phi\right]^2 . \tag{2}$$

The nonuniformity of the magnetic field (~ 1 Oe along the length of the chamber) is small in comparison with both the field itself (~ 100 Oe) and the dipole energy (~ 50–80 Oe), so that F can be minimized in two steps. We first find the minimum of the sum of the Zeeman and dipole energies, assuming that the Larmor frequency is independent of the coordinate. This minimization leads us to the condition

$$\cos\phi = \left(\frac{1}{2} - \cos\beta\right) \Big/ \left(1 + \cos\beta\right). \tag{3}$$

The only limitation on the spatial distribution of β comes from the requirement $\int S_z dV = \int S\cos\beta dV = \text{const}$, i.e., that the minimum be degenerate. In the next step we minimize the coordinate-dependent part of the Zeeman energy along with the gradient energy G. Here we seek the minimum among states for which ϕ and β are

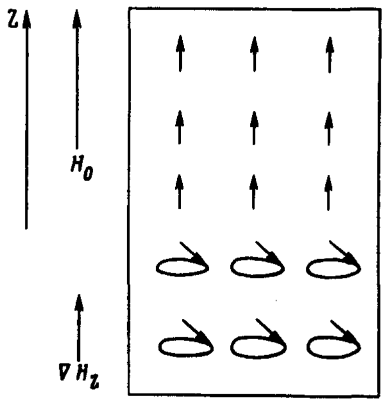

FIG. 1.

related with condition (3). Clearly, the energy F decreases with increasing longitudinal component of the spin in the region of the stronger field, because of a decrease in this energy in the region of the weaker field. This redistribution of the spin can easily be arranged by virtue of the flow of superfluid spin currents. The redistribution continues until a structure consisting of two regions forms (Fig. 1). At values of z above a certain z_0—chosen to conserve $\int S_z dV$—the spin is parallel to the field ($\beta = 0$), while at $z < z_0$ we have $\beta \approx \theta_0 = \arccos(-1/4)$. A further increase in β at $z < z_0$ leads to an increase in the dipole energy and is thus not favorable. The thickness of the boundary region, λ, is determined from the equality in order of magnitude of the gradient and variable Zeeman energies: $c^2/\omega_L \lambda^2 \sim \lambda \nabla \omega_L$, i.e., $\lambda^3 \sim c^2/\omega_L \nabla \omega_L$, where c is the velocity of spin waves. With $H_0 = 150$ Oe and $\nabla H_0 \sim 1$ Oe/cm we find $\lambda \sim 10^{-2}$ cm.

When we actually go through this minimization procedure, we find the Lagrange equation $(d/d\xi)(\partial \mathscr{L}/\partial u') - (\partial \mathscr{L}/\partial u) = 0$ with the Lagrangian

$$\mathscr{L} = \frac{2u+3}{(1-u)(3+u)(4u+1)} \left(\frac{du}{d\xi} \right)^2 + \xi(1-u), \tag{4}$$

where $u = \cos\beta$, and where we have introduced the dimensionless coordinate $\xi = z/\lambda$. Figure 2 shows the numerical solution of this equation with the boundary conditions $u \to 1$ as $\xi \to \infty$ and $u \to -1/4$ as $\xi \to -\infty$. A spin precesses at the same frequency

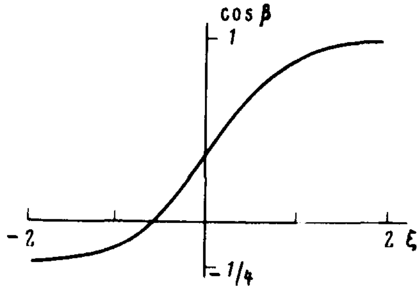

FIG. 2. Change in $\cos\beta$ along the domain wall separating the regions with $\beta = \theta_0$ and $\beta = 0$.

over the entire chamber volume; this frequency is the Larmor frequency at the position of the domain wall.

The test chamber in Ref. 3 was an essentially closed volume, so that spin currents of the type discussed here could not flow under steady-state conditions. The condition for the absence of a flux S_z, which is found in the standard way for functional (1), is

$$(3+u)\frac{d\alpha}{d\xi} - \frac{d\phi}{d\xi} = 0; \tag{5}$$

since we have $\phi = \text{const}$ far from the domain wall, we have $\alpha = \text{const}$ by virtue of (5). In other words, the spins precess with the same phase. It is this precession that creates the long-lived induction signal.

The duration of the long-lived signal is determined by spin relaxation processes. The relaxation results in an increase in the size of the region with $\mathbf{S}\|\mathbf{H}_0$, and the domain wall shifts toward the weak-field region, explaining the decrease in the frequency of the long-lived signal over time found in the experiments of Ref. 3. Two mechanisms are responsible for the relaxation of this solution: the mechanism of Leggett and Takagi,[6] which is important to the extent that the precession frequency differs from the local Larmor frequency, and spin diffusion through the domain wall. An elementary calculation of the energy balance leads to the following equation for the time evolution of the precession frequency ω, reckoned from its smallest value in the chamber volume:

$$\frac{d\omega}{dt} = -\frac{4}{5}\frac{D}{\lambda}\sigma(\nabla\omega_L) - \frac{1}{4}\tau_{LT}\omega^3 . \tag{6}$$

Here D is the spin diffusion coefficient, τ_{LT} is the characteristic time for the "internal" relaxation mechanism, and the number $\sigma \sim 1$ depends on the shape of the domain wall (in our case, $\sigma \approx 1.10$). As equilibrium is approached, the second term on the right side of (6) approaches zero, and the rate of change of the frequency approaches a constant. Determining this constant, we can find the experimental value of the combination $D/c^{2/3}$.

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[1]L. R. Corruccini and D. D. Osheroff, Phys. Rev. B **17**, 126 (1978).
[2]R. W. Giannetta, E. N. Smith, and D. M. Lee, J. Low Temp. Phys. **45**, 295 (1981).
[3]A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev, and Yu. M. Mukharskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. This Issue, p. 256. [JETP Lett. **40**, 1040 (1984)].
[4]A. J. Leggett, Ann. Phys. **85**, 11 (1974).
[5]I. A. Fomin, Zh. Eksp. Teor. Fiz. **78**, 2392 (1980) [Sov. Phys. JETP **51**, 1203 (1980)].
[6]A. J. Leggett and S. Takagi, Ann. Phys. **106**, 79 (1977).

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