de Sitter initial state of the universe as a result of the asymptotic disappearance of gravitational interactions of matter

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(Submitted 10 August 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 6, 265–268 (25 September 1984)

In the "effective" action for the gravitational field and matter, the "constant" of the coupling of the matter with gravitation is a function of the energy density. In this case a Λ-shaped term automatically arises in the Einstein equations. The particular conditions under which the initial state of the universe turns out to be a de Sitter state are analyzed.

Ad hoc modified Einstein equations which describe a de Sitter universe of Planckian dimensions at high energy densities have been analyzed in several studies.[1–4] Those equations were not derived from a variational principle. In the present letter we show

 0021-3640/84/181043-05$01.00

that hydrodynamic equations which are analogous to those of Refs. 1–4 and which can effectively describe the universe in its early moments can be found from the action

$$S = \frac{c^4}{16\pi G_0} \int (R + 2\kappa\epsilon)\sqrt{-g}\, d^4x , \tag{1}$$

if κ is assumed to be a function of the energy density ϵ: $\kappa \equiv \kappa(\epsilon)$. A dependence of κ on ϵ can effectively describe the physical processes that occur at Planckian densities.[5] Varying action (1) with respect to the metric g_{ik}, we find the equation

$$R^i_k - \frac{1}{2}\delta^i_k R = G(\epsilon)\widetilde{T}^i_k + \Lambda(\epsilon)\,\delta^i_k . \tag{2}$$

Here $\widetilde{T}^i_k = (\epsilon + p)u^i u_k - p\delta^i_k$, and the functions $G(\epsilon)$ and $\Lambda(\epsilon)$ are expressed in terms of $\kappa(\epsilon)$ by

$$G(\epsilon) = \epsilon\frac{\partial\kappa}{\partial\epsilon} + \kappa , \quad \Lambda(\epsilon) = -\epsilon^2\frac{\partial\kappa}{\partial\epsilon} . \tag{3}$$

In varying action (1) we use Fok's procedure,[6] introducing particles with a density n which satisfies the continuity equation[1)] $(nu^i)_{;i} = 0$.

In the general case of an arbitrary function $\kappa(\epsilon)$ the energy-momentum tensor found from action (1) contains a Λ-shaped term. This term vanishes if and only if this action is the same as the Einstein-Fok action, i.e., if and only if $\kappa = \text{const}$.

The function $\psi(\epsilon) = c^4\kappa(\epsilon)/8\pi G_0$ must be a function of the dimensionless energy density $x = \epsilon/\epsilon_0$, where the only candidate for the role of ϵ_0 is a Planckian density.

In the theories of Refs. 1–4 a boundedness of $G(\epsilon)\widetilde{T}^i_k$ was adopted as a fundamental law in all versions of the concept.[1] In several cases, a boundedness of the "energy" density ϵ emerges from the equations. This is the case, for example, in the model of Ref. 3 for a closed isotropic universe. However, there is apparently no reason of any sort to exclude the case $\epsilon \to \infty$ from this formalism if we can satisfy the condition $G(\epsilon)\widetilde{T}^i_k < \text{const}$ in this case.

In accordance with the ideas of Refs. 1–4, our problem is to find, through a variation of generalized action (1), modified equations (2) which describe a universe that arbitrarily becomes close to a de Sitter universe at large values of ϵ. We formulate the result as a theorem: If $G(\epsilon)$ in Eqs. (2) is positive for arbitrary ϵ, and if $G(\epsilon)\epsilon \to 0$ in the limit $\epsilon \to \infty$, then a homogeneous and isotropic closed universe can be described at high densities by a solution that is arbitrarily close to the de Sitter solution. The condition $G(\epsilon)\epsilon \to 0$ in the limit $\epsilon \to \infty$ in a sense means an asymptotic degeneracy of the gravitational interactions of matter. Under the natural requirement $G(\epsilon) \geqslant 0$ for arbitrary ϵ, it would be impossible to arrange a simultaneous vanishing of $G(\epsilon)\epsilon$ and $\Lambda(\epsilon)$ with increasing ϵ.

Let us prove this theorem. We first note that the functions $G(\epsilon)$ and $\Lambda(\epsilon)$ are related by

$$\Lambda(\epsilon) = \int_0^\epsilon G(\epsilon)\, d\epsilon - G(\epsilon)\epsilon \tag{4}$$

Since $G(\epsilon)\epsilon \to 0$ as $\epsilon \to \infty$, and $G(\epsilon) > 0$, it follows from (4) that with increasing density ϵ the Λ-shaped term tends toward a constant value $\int_0^\infty G(\epsilon) d\epsilon$.

We consider a closed isotropic universe with the metric

$$ds^2 = dt^2 - a^2(t)\gamma_{\alpha\beta} dx^\alpha dx^\beta , \tag{5}$$

where $\gamma_{\alpha\beta}$ is the metric of a 3-space of constant positive curvature. Equations of motion for matter are found from the conditions $T^i_{k;i} = 0$, which are consequences of the Bianchi identities. In an isotropic universe with metric (5), the equation $[G(\epsilon)\widetilde{T}^i_0 + \Lambda(\epsilon)\delta^i_o]_{;i} = 0$ becomes

$$d\epsilon = -3(p + \epsilon) d \ln a. \tag{6}$$

Action (1) is written

$$S = -\frac{3\pi c^4}{4G_0} \int \left[\dot{a}^2 a - a + \frac{a^3}{3} \int_0^\epsilon G(\epsilon) d\epsilon \right] dt . \tag{7}$$

The corresponding Hamiltonian is

$$\mathcal{H} = -\frac{G_0}{3\pi c^4} \frac{P^2}{a} + \frac{3\pi c^4 a}{4G_0} (-V(a) - 1), \tag{8}$$

where $P = (3\pi c^4/2G_0) a \dot{a}$, and the potential $V(a)$ is

$$V(a) = -\frac{a^2}{3} \int_0^{\epsilon(a)} G(\epsilon) d\epsilon . \tag{9}$$

To find the evolution of the scale factor a we use (6) along with the 0-0 component of Einstein's equations, which reduces to a condition on the Hamiltonian $\mathcal{H}$:$\mathcal{H} = 0$. We find then

$$\dot{a}^2 + V(a) = -1. \tag{10}$$

Our problem is thus equivalent to the problem of the motion of a particle with an energy of -1 in a field with a potential $V(a)$. We will see below that here there is only a single allowed region of motion, in contrast with the situation in Ref. 7. As an example we consider a dust with $p = 0$. From (6) we then find $\epsilon = c/a^3$. To find the potential $V(a)$ in the asymptotic limits ($\epsilon \to 0$, $\epsilon \to \infty$), for a broad range of functions $G(\epsilon)$ satisfying the conditions given above, we find $a_{\min}$ and $a_{\max}$ (the minimum and maximum radii of the universe) from the condition $\dot{a}_{\substack{\min \\ \max}} = 0$ or, equivalently,

$$V(a_{\substack{\min \\ \max}}) = -1. \tag{11}$$

For $\epsilon \ll \epsilon_0 \sim \epsilon_{Pl}$ [the radius of the universe at $\epsilon \sim \epsilon_{Pl}$ is $a \sim (a^2_{\min} a_{\max})^{1/3}$; Ref. 3] we have $G(\epsilon) \simeq 8\pi G_0/c^4$, and the integral in (19) is $\int_0^\epsilon G(\epsilon) d\epsilon \simeq (8\pi G_0/c^4)\epsilon(a)$. Using (11), we then find

$$V(a) \simeq -\frac{a_{\max}}{a} , \qquad a \gg (a^2_{\min} a_{\max})^{1/3} .$$

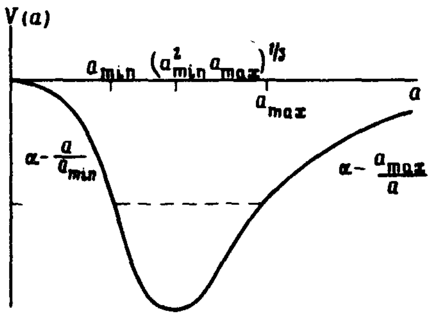

FIG. 1. The potential $V(a)$ as a function of the scale factor a.

For $\epsilon \gg \epsilon_{Pl}$, we correspondingly have $\int_0^\epsilon G(\epsilon) d\epsilon \sim \frac{8\pi G_0}{c^4}\epsilon_{Pl} \sim$ const and

$$V(a) \approx -\left(\frac{a}{a_{\min}}\right)^2, \quad a \ll (a_{\min}^2\, a_{\max})^{1/3}.$$

The potential $V(a)$ is sketched in Fig. 1. At $\epsilon \ll \epsilon_{Pl}$ the universe obviously evolves in the manner of a Friedman universe filled with ordinary dust, while at $\epsilon \gg \epsilon_{Pl}$ the universe is described by the de Sitter solution. As an example we find the value of the radius a_{min} for $G(\epsilon) = (8\pi G_0/c^4)(1 + {}^{\epsilon}/_{\epsilon_{Pl}})^{-2}$. In this case Eq. (11) becomes

$$\frac{8\pi G_0}{3c^2}\, a^2_{\substack{\min\\ \max}}\, \frac{M m_{Pl}}{M l_{Pl}^3 + m_{Pl}\, a^3_{\substack{\min\\ \max}}} = 1, \tag{12}$$

where M is the total mass of the universe. For $Ml^3 \gg m_{Pl}\, a^3$ we find $a_{\min} \sim (\sqrt{3}/8\pi) l_{Pl}$. In other words, if the total mass of a dusty universe is much greater than the Planckian mass, the universe rebounds when it contracts to Planckian dimensions in the course of the collapse. The proximity to Planckian dimensions is determined by the total mass of dust in the universe.[3] The final state of a collapsing universe doubles as the initial state of the next expansion cycle. In turn, the problems of a perpetually oscillating universe are solvable when quantum fluctuations are taken into account at the time at which the collapse stops, at the minimum radius of the universe.[3]

We note in conclusion that the possibility of avoiding a singularity in this case stems from the violation of the condition of energy dominance at high densities. As was shown in Ref. 3, a dependence of the mass of the particles on the energy density can lead to such a situation. It may be that the modification of action (1) results from the same factors.

[1)] Figuratively speaking, the particles do not disappear in the course of the collapse; instead, each of the particles becomes progressively more "incorporeal" (less massive).

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Translated by Dave Parsons.
Edited by S. J. Amoretty