Dispersion echo and pulse self-contraction

V. I. Reshetskiĭ

Pacific Oceanological Institute, Far East Scientific Center, Academy of Sciences of the USSR

(Submitted 5 June 1984)

Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 7, 271–273 (10 October 1984)

A method is proposed for reversing the spreading of a packet of plane waves. It may be possible to use this effect to increase the range over which signals can propagate through an absorbing medium.

Since the first studies of the spin echo carried out by Hahn,[1] many similar studies have been made of the coherent dynamics of many-body systems. Optical, electroacoustic, cyclotron, plasma, etc., echo phenomena have been discovered, and many have evolved into independent fields of research with extensive work on applications.[2,3]

In the present letter we report a study of a reversal of the spreading of a packet of dispersing waves as a result of a parametric interaction with a pump wave. We call the effect the "dispersion echo."

We consider a one-dimensional boundary-value problem. We assume that the dispersive medium occupies the half-space $z>0$ and that an electromagnetic input signal is given at the boundary of this medium[4]:

$$\mathbf{E}_1(t, 0) = \frac{1}{2}\mathbf{e}_1 \int_0^{\infty} A_1(\omega)\, e^{i\omega t} d\omega + \text{ c.c.}, \tag{1}$$

As this signal propagates through the dispersive medium, it is described by[4]

$$\mathbf{E}_1(t, z) = \frac{1}{2}\mathbf{e}_1 \int A_1(\omega) e^{iz\psi(\omega)}\, d\omega + \text{ c.c.}, \tag{2}$$

where $\psi(\omega) = \left[\omega\frac{t}{z} - k(\omega)\right]$, and $k(\omega) = \omega/v(\omega)$. Expanding the phase velocity in a series near the extremum,

$$v(\omega) = v(\omega_e + \Delta\omega) \simeq v(\omega_e) + \frac{1}{2}(\partial^2 v/\partial\omega^2)_{\omega_e}(\Delta\omega)^2,$$

we find

$$\psi(\omega) \simeq v^{-2}(\omega_e)\omega_e(\partial^2 v/\partial\omega^2)_{\omega_e}(\Delta\omega)^2, \tag{3}$$

where the point $\omega = \omega_e$ is found from the condition $\partial v/\partial\omega = 0$; i.e., at this point the phase velocity reaches a maximum or minimum value. There are usually two such points (see Ref. 5, for example), as follows from the dispersive behavior of the refractive index. We will assume below that the second point lies outside the spectral region of the pulse. In deriving expression (3) we used a coordinate system moving with an observer at the extreme phase velocity $v(\omega_e)$, i.e., $z/t = v(\omega_e)$. We know[4] that when the phase $\psi(\omega)$ has stationary points the pulse (2) decays asymptotically over the distance (z) that it traverses: $\mathbf{E}_1(t, z\to\infty) \sim (z)^{-1/2}$ (we are ignoring dissipation). In the absence of stationary points of the phase, the decay is significantly more rapid: $\mathbf{E}_1(t, z\to\infty) \sim (z)^{-2}$. The reason for the decay in these cases is the dispersion, which causes some of the plane waves to lag behind the group velocity and causes others to move ahead of it. As a result, there is a self-extinction of the pulse by interference; i.e., the temporal coherence of signal (1) is lost. This decay process is evidently of a reversible nature and is analogous to the decay of the dipole moment of a coherent ensemble of a many-body system due to inhomogeneous broadening.[1] To restore the coherence of the signal, we send it through a nondispersive medium with a quadratic nonlinearity that fills the space $z_2 \geqslant z \geqslant z_1$, in which a monochromatic pump wave

$$\mathbf{E}_2 = \frac{1}{2}\mathbf{e}_2 A_2 e^{i(2\omega_e t - 2k_e z)} + \text{c.c.}, \tag{4}$$

is propagating. We are interested in the evolution of the signal wave E_1, on which wave (4) is acting. The corresponding closed system of equations is[6]

$$\frac{dA_1(\omega, z)}{dz} = -i\sigma_1 A_1^*(2\omega_e - \omega, z),$$
$$\frac{dA_1^*(2\omega_e - \omega, z)}{dz} = i\sigma_2 A_1(\omega, z) \tag{5}$$

where

$$\sigma_1 = \mathbf{e}_1\frac{2\pi\omega(\chi : \mathbf{e}_1\mathbf{e}_2)A_2}{cn}, \qquad \sigma_2 = \mathbf{e}_2\frac{2\pi(2\omega_e - \omega)(\chi : e_1 e_2)A_2^*}{cn},$$

$$\sigma_1\sigma_2 = |\sigma|^2, \quad |\sigma| > 0, \quad z_2 - z_1 = L$$

and χ, c, and n are respectively the quadratically nonlinear susceptibility, the velocity of light in a vacuum, and the refractive index. In deriving the system of truncated equations (5), we used the method of a slowly varying amplitude. Under the condition $|A_2|\gg|A_1|$, system (5) has a solution in the approximation of a given pump wave:

$$\mathbf{E}_1(t, z \geq z_2) = \frac{1}{2}\mathbf{e}_1 \operatorname{ch}|\sigma|L \int_0^\infty A_1(\omega, z_1) e^{i(z-z_1)\psi(\omega)} d\omega$$

$$- i\mathbf{e}_1 \frac{\sigma_1}{|\sigma|} \operatorname{sh}|\sigma|L \int_0^\infty A_1^*(2\omega_e - \omega, z_1) e^{i(z-z_1)\psi(\omega)} d\omega + \text{c.c.}. \qquad (6)$$

According to (1) and (2), the boundary conditions are

$$A_1(\omega, z = z_1) = A_1(\omega, 0)\exp[iz_1\psi(\omega)],$$

$$A_1^*(2\omega_e - \omega, z = z_1) = A_1^*(2\omega_e - \omega, 0)\exp[-iz_1\psi(2\omega_e - \omega)].$$

At a sufficiently large value of z_1, the first term in solution (6) continues to decay after the interaction, while the second tends to recover its original form, (1), at the point $z = z_0$, whose position must depend on the frequency and can be determined by equating the overall phase of the second term: $\psi(\omega)(z_0 - z_1) - \psi(2\omega_e - \omega)z_1 = 0$. Using (3), we find

$$z_0 = z_1\left[1 + \frac{\psi(2\omega_e - \omega)}{\psi(\omega)}\right] = 2z_1.$$

At the point z_0 the pulse has therefore undergone a self-contraction to its original shape, as in Eq. (1). It is important to note that pump frequency (4) must correspond exactly to the frequency at which the phase velocity is at a maximum. Otherwise, it would be necessary to consider the term $(\partial v/\partial\omega)\Delta\omega$ in expansion (3). It is easy to show that in this case the coherence of signal (1) is not restored, and the spreading occurs twice as fast.

This effect may find use in increasing the range over which signals can propagate through absorbing media, since the rates of most of the processes responsible for the damping (absorption, for example) are proportional to $(\mathbf{E}_1)^n$, where $n = 1, 2, \ldots$. If the pulse is first passed through a dispersive medium, so that $\mathbf{E}_1 \to 0$, and then sent through the absorbing medium, it will traverse the absorbing medium with essentially no loss of energy. The pulse should then be subjected to the nonlinear conversion described above and passed again through the same dispersive medium. As a result, the original shape of the pulse will be restored.

[1]E. L. Hahn, Phys. Rev. **80**, 580 (1950).
[2]A. Korpel and M. Chatterjee, Proc. IEEE **69**,1539 (1981).
[3]M. S. Gitlin and L. A. Ostrovskiĭ, Zh. Eksp. Teor. Fiz. **85**, 487 (1983) [Sov. Phys. JETP **58**, 285 (1983)].
[4]G. B. Whitham, Linear and Nonlinear Waves, Wiley-Interscience, New York, 1974.
[5]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Nauka, Moscow, 1982.

Translated by Dave Parsons
Edited by S. J. Amoretty