Chains of Rossby solitons and gradient solitons

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The two-dimensional nonlinear equation $\nabla_\perp^2\varphi - \varphi + \varphi^2 = 0$, which describes Rossby waves and nonlinear drift waves in a plasma in a magnetic field, has an approximate solution consisting of an infinite chain of solitons that are localized in the transverse direction.

It has been shown theoretically[1,2] that a circular Rossby-wave soliton exists in a rotating shallow liquid of finite depth. This soliton is described by the nonlinear equation

$$\nabla_\perp^2\varphi - \varphi + \varphi^2 = 0. \tag{1}$$

Here φ is a measure of the deviation of the depth of the liquid from its unperturbed value, and $\nabla_\perp$ is the gradient operator in the horizontal plane. The function φ and the coordinates are assumed to have been rendered dimensionless in an appropriate manner. Nezlin *et al.*[3] have experimentally observed the excitation of a circular Rossby-wave soliton of this sort by certain external factors. In other experiments under conditions corresponding to a Kelvin-Helmholtz instability,[4] the same investigators observed a nonlinear structure which may be interpreted as a chain of Rossby solitons moving in succession ("marching in column formation") along the symmetry direction of the system, i.e, along the azimuthal direction in the apparatus (see Fig. 2 in Ref. 4). It is interesting in this connection to determine whether Eq. (1) describes, in addition to an isolated circular soliton, a chain of solitons of the sort observed in Ref. 4. We show in this letter that this is the case.[1)]

Since we cannot find an exact analytic solution of two-dimensional equation (1), we find an approximate solution, assuming

$$\varphi(x, y) = \varphi_0(x) + \varphi_1(x)\cos ky. \tag{2}$$

Here k is an unknown parameter, φ_0 and φ_1 are unknown functions satisfying $\hat{\varphi}_1 < \varphi_0$, x is the direction of the nonuniformity, and y is the symmetry direction of the unperturbed state. Using (2) and this inequality, we find from (1) the following equations for φ_0 and φ_1:

$$\varphi_0'' - \varphi_0 + \varphi_0^2 = 0, \tag{3}$$

$$\varphi_1'' - (1 + k^2)\varphi_1 + 2\varphi_0\varphi_1 = 0, \tag{4}$$

where the prime means a derivative with respect to x. A solution of (3), which is bounded as $|x| \to \infty$, is well known (see Ref. 8, for example):

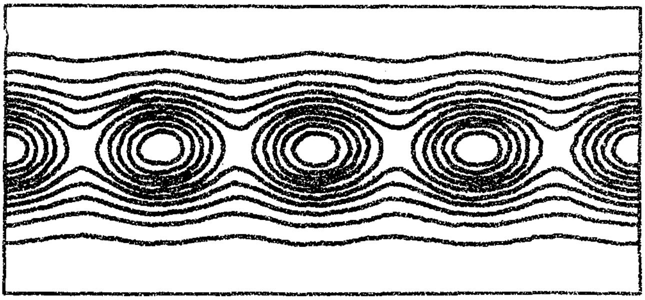

FIG. 1.

$$\varphi_0 = \frac{3}{2}\cosh^{-2}(x/2). \quad (5)$$

Substituting (5) into (4), and requiring the φ_1 be bounded in the limit $|x| \to \infty$, we find (cf. Ref. 9)

$$\varphi_1 = C\cosh^{-3}(x/2), \quad k = \sqrt{5}/2, \quad (6)$$

where C is a constant. It follows from (2), (4), and (5) that the solution in which we are interested is of the form

$$\varphi = \frac{3}{2}\cosh^{-2}(x/2)\left[1 + \frac{\epsilon\cos(\sqrt{5}\,y/2)}{\cosh(x/2)}\right], \quad (7)$$

where ϵ is an arbitrary small parameter.

Lines of $\varphi = \text{const}$ are sketched in Fig. 1. In accordance with Refs. 1 and 2, the φ contour lines coincide with lines of the perturbed motion of the liquid. This figure thus shows that solution (7) describes chains of Rossby solitons similar to those observed in Ref. 4. This result should not, however, be taken as a solid interpretation of the experiments of Ref. 4, since our original equation, (1), does not incorporate the Kelvin-Helmholtz instability which occurred under the conditions of Ref. 4.

An equation of the type in (1) also describes a rather broad class of nonlinear gradient waves in a plasma in a magnetic field, including so-called drift waves.[8] It has been shown elsewhere that waves of this sort may be manifested as circular (cylindrically symmetric) drift solitons. Petviashvili[8] has also discussed the question of one-dimensional drift solitons describable by an electric potential (5); he pointed out that such solitons are unstable with respect to perturbations that are nonuniform along y. The nonlinear soliton-chain structure which we have observed, (7), can apparently arise from the instability analyzed in Ref. 8 and also from other instabilities.

The analogy between the equation for Rossby waves and that for nonlinear drift waves and the experimental results of Ref. 4 regarding a chain of Rossby solitons suggest that chains of solitons should also be observed in corresponding experiments on drift waves. Of interest in this connection is a comment by N. S. Buchel'nikova (private communication) that drift solitons may have occurred in Q-machine experiments,[10] but the question was not pursued at the time.

[1)] Zaĭtsev[5] and Zhdanov and Trubnikov[6] have also examined chains of solitons of this sort in situations described by a Kadomtsev-Petviashvili equation.[7]

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