Valley degeneracy of two-dimensional electrons on the (111) surface of silicon

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The degree of valley degeneracy of two-dimensional electrons on the (111) surface of silicon in the absence of a magnetic field is measured directly for the first time. It is found that the degree of valley degeneracy is six, in contrast to two, which was obtained by measuring the Shubnikov–de Haas oscillations.

In the physics of two-dimensional electronic systems, the problem of the valley degeneracy of two-dimensional electrons on the (111) surface of silicon is one of the oldest and thus far unsolved problems. This problem arose almost ten years ago after the first measurements of Shubnikov oscillations in a two-dimensional electron gas on

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the (111) surface of silicon.[1,2] The degree of valley degeneracy, found in these experiments from the period of oscillations, turned out to be equal to two, instead of six, as expected from the theoretical predictions based on the effective-mass approximation.[3] This paradox gave rise to a series of models,[4] none of which convincingly resolved it. Several years later, it was found that after several special treatments of the silicon surface and before it becomes oxidized, it is possible to obtain samples with $g_v = 6$.[5] This circumstance compounded the uncertainty of this problem, which has not been solved to this day.[6,7]

The difficulty posed by this problem involved not only purely theoretical questions but also the fact that the measurements of the Shubnikov–de Haas oscillations were the only method used to determine the degree of valley degeneracy. It is therefore of great interest to determine g_v by a different method, which is not related to magnetic measurements.

Such an opportunity arose in the study of the phonon-entrainment of two-dimensional electrons into the inversion layers on the (001) surface of silicon.[8] This method can be summarized as follows: In a two-dimensional system the phonon-electron interaction has a sharp singularity when the wave vector of the phonon $q_{\rm ph}$ is equal to twice the Fermi vector k_F, i.e., when

$$q_{\rm ph} = 2k_F \tag{1}$$

(see Ref. 8 and Ref. 2 cited therein). If in the MIS sample the surface charge density N is fixed by measuring the gate voltage V_g, then the singularity at $q_{\rm ph} = 2k_F$ will be manifested as a peak in the curve of $(\alpha_{\rm ph}\kappa^{-1})$ versus the temperature, when the value of $q_{\rm ph}$, which corresponds to the maximum of the thermal distribution of phonons, is equal to $2k_F$ ($\alpha_{\rm ph}$ is the thermo-emf associated with the phonon entrainment of charges, and κ is the thermal conductivity of the sample). Since the quantity k_F is related to the surface charge density by

$$N = 2g_v \pi k_F^2 (2\pi)^{-2} \tag{2}$$

(for a spherical Fermi surface), and since the quantity $q_{\rm ph}$ is related to the temperature of the sample by

$$q_{\rm ph} = 4{,}96\,\frac{k}{\hbar}\,\frac{T}{\bar{u}} \simeq 1{,}2 \cdot 10^{6}\ \mathrm{T/cm} \tag{3}$$

where $\bar{u}$ is the mean velocity of the phonons, and the factor 4.96 is the coefficient in Wien's displacement law, we can determine the value of g_v from relations (1), (2), and (3).

For the experiment, we used MIS structures fabricated using the standard technology on surfaces close to the (111) surface [8°30′ to (111)]. At 4.2 K the maximum carrier mobility is 3000 cm^2 (V·s)$^{-1}$. The thermo-emf was measured in the interval 0.4–9 K, using the procedure described previously.[9]

It has been established elsewhere that the thermo-emf of the charges is due to the diffusion of electrons, α_e, and their entrainment by phonons, $\alpha_{\rm ph}$. The diffusion part, which is proportional to the temperature, can be singled out by extrapolating the curve

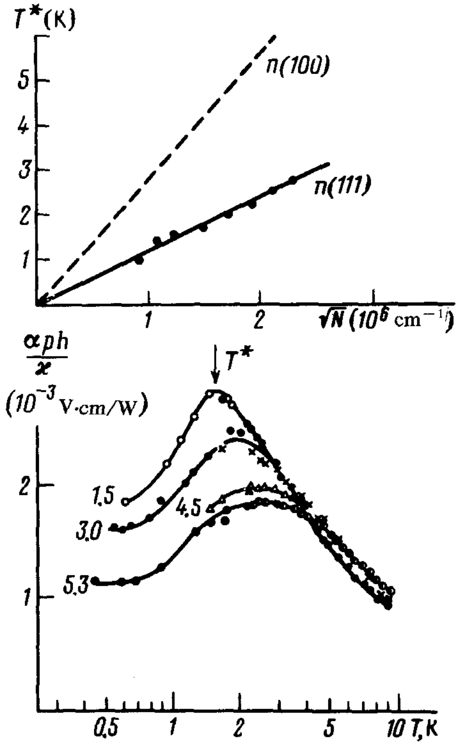

FIG. 1. Plot of the $\alpha_{ph}\kappa^{-1}$ versus T(K). The parameter of the curves is the electron density N (in units of 10^{12} cm^{-2}) ○, ⊗, ⊙ are measurements in a cryostat with a low-temperature He^3 bath ($T = 0.3$ K); ●, ◑, ×, ▲ are measurements in an ordinary He^4 cryostat. The arrow marks the temperature T^*. The plot of T^* versus $N^{1/2}$ is shown at the top. The broken line is the same plot for the structure n(100), for which $g_v = 2$.[8]

of α/T versus T^2 to zero temperature. The value of α_e was determined in this manner to within $\simeq$10% for each value of the density N. For densities ranging from 1.5 to 5.3×10^{12} cm^{-2} we found that $\alpha_e T^{-1}$ varies from 10^{-5} to 2.6×10^{-6} V·K^{-2}. These values were used to refine the values of α_{ph}. The correction to the contribution from α_e near the maxima of $\alpha_{ph}\ \kappa^{-1}$ did not exceed 10% of α_{ph}.

The temperature dependence of $\alpha_{ph}\kappa^{-1}$ is shown in Fig. 1. Also shown in this figure is the T^* versus $N^{1/2}$ plot—the temperature at which the peak in $\alpha_{ph}\kappa^{-1}$ occurs. We see that T^* is directly proportional to $N^{1/2}$, as expected from the equations presented above. However, the slope of the straight line T^* ($N^{1/2}$) differs substantially (by a factor of 2.4) from the slope[8] obtained previously for structure on (100) surfaces, where $g_v = 2$. The change in the slope of the straight line T^* ($N^{1/2}$) indicates that the value of g_v for the (111) samples studied is substantially larger than 2.

It is more complicated to calculate the degree of valley degeneracy from the phonon entrainment on the (111) surface of silicon, because the Fermi surface in the effective-mass approximation[3] in this case is comprised of six ellipses, whose axes differ by a factor of 1.9. Equation (2) in this case becomes

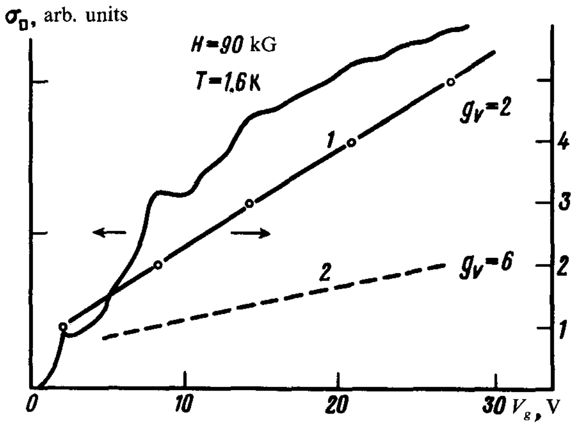

FIG. 2. Plot of the conductivity $\sigma_\square$ versus the gate voltage V_g. The number of Shubnikov oscillations is plotted along the Y axis on the right. The broken line corresponds to $g_v = 6$.

$$N = 2g_v \pi k_1 k_2 (2\pi)^{-2} ,$$

where k_1 and k_2 are the semiaxes of the Fermi ellipse. In the case of a nonspherical surface, it is not obvious what value of k_F corresponds to the condition (1). In the experiments the heat flowed parallel to the minimum dimension of the two valleys. Clearly, this dimension must be incorporated into Eq. (1). In this case, we find that $g_v = 6$. The assumption that the peak in $(\alpha_{\text{ph}} \kappa^{-1})$ corresponds to the larger value of k_F increases the computed value of g_v.

The dependence of the conductivity $\alpha_\square$ on the gate voltage in a magnetic field of 90 kG was measured for the same samples (Fig. 2). The slope of the straight line 1, which is proportional to the period of the observed oscillations, corresponds to $g_v = 2 \pm 0.1$. The straight line 2, which corresponds to $g_v = 6$, is shown for comparison.

The results presented above show that in the absence of a magnetic field, the degree of valley degeneracy of electrons on the (111) surface of silicon is equal to six, in agreement with the predictions based on the effective-mass approximation, rather than two, as follows from the measurements of the Shubnikov oscillations.

The characteristic behavior of two-dimensional electrons in quantizing magnetic fields must be studied further in order to resolve this discrepancy.

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