Kapitsa jump in a gas of spin-polarized atomic hydrogen

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The Kapitsa jump at the interface of a gas of spin-polarized atomic hydrogen and liquid helium is analyzed. At any temperature the dominant mechanism for energy exchange between the gas and the surface is an adsorption mechanism. Estimates from the results derived here show that the increase in the Kapitsa jump with decreasing temperature poses serious obstacles to reaching the region of Bose condensation.

1. It has been shown theoretically[1,2] and experimentally[3–5] that a decay mechanism involving three-body dipole recombination and a change in spin configuration unavoidably operates in a gas of spin-polarized atomic hydrogen. This process and phenomenon of the thermal explosion[6] (the thermal explosion was observed experimentally in Refs. 3 and 5) impose serious restrictions on the gas density that can be obtained; a realistic upper limit is $n \sim 10^{19}$ cm^{-3}. The problem of reaching the region of Bose condensation is thus unavoidably linked with the need to substantially lower the temperature. The question of the Kapitsa temperature jump at the interface between a gas and the surface of liquid helium thus becomes a question of fundamental importance.

Two mechanisms typically operate to transfer energy from a gas to a helium surface. The first, involving an inelastic scattering of hydrogen atoms by the surface, was studied in Ref. 7. At low temperatures, however, this mechanism is of negligible importance in comparison with the second mechanism, which involves an energy transfer as atoms are captured into an adsorption well. The basic reason for this situation is that the wave function of the particle in the final state is localized in this second case. It is important to note that this assertion remains in force when the surface is completely covered by adsorbed atoms.

As we will see in the results below, the increase in the Kapitsa jump with decreasing temperature poses serious obstacles to reaching the region of Bose condensation.

2. Let us examine the mechanism for energy exchange between a gas and a liquid helium surface which involves the adsorption and desorption of hydrogen atoms, accompanied by the emission and absorption, respectively, of capillary waves: ripplons. The interaction of the hydrogen atom with ripplons is described by the operator

$$\hat{V} = \frac{dU(z)}{dz}\,\hat{\zeta}(\vec{\rho}), \tag{1}$$

where $U(z)$ is the static interaction potential of the atom with the helium surface. This potential has a well of depth ~ 5 K, in which there is only a single discrete level, with a binding energy $\epsilon_0 \approx 1$ K (Ref. 8). The radius vector of the atom is $\mathbf{R} \equiv \{\vec{\rho}, z\}$, and z is the projection of this vector on the normal to the surface. The operator representing the displacement of surface points along the z axis can be written in the following form in second quantization:

$$\hat{\zeta}(\vec{\rho}) = \sum_{\mathbf{q}} \left(\frac{\hbar q}{2\rho_0 \omega S}\right)^{1/2} (\hat{c}_q e^{i\mathbf{q}\vec{\rho}} - \hat{c}_q^{+} e^{-i\mathbf{q}\vec{\rho}}) . \tag{2}$$

Here the operators c_q^+ and c_q represent the emission and absorption of a ripplon,

$$\omega = (\sigma/\rho_0)^{1/2} q^{3/2} \tag{3}$$

is the ripplon frequency, ρ_0 is the density of the helium, σ is the surface-tension coefficient, and S is the area of the surface. The energy and momentum conservation laws are

$$\epsilon_k = \hbar\omega(q) + \epsilon_f - \epsilon_0 , \tag{4}$$

$$\mathbf{k}_{\parallel} = \mathbf{f} + \mathbf{q} , \tag{5}$$

where $\mathbf{k} \equiv \{\mathbf{k}_{\parallel}, k_z\}$ and ϵ_k are the momentum and kinetic energy of the free motion of a hydrogen atom in the volume, and $\mathbf{f}$ and ϵ_f are the corresponding properties at the surface. The energy flux from the gas to the surface is

$$J = \int \frac{d^3k}{(2\pi)^3} \frac{d^2q}{(2\pi)^2} \frac{\hbar k_z}{m} \epsilon_k W(\mathbf{k},\mathbf{q})$$

$$\times \{ n_k(T)(1+n_q(T_0))(1+N_f(T_0)) - (1+n_k(T))n_q(T_0)N_f(T_0) \} ; \tag{6}$$

$$W(\mathbf{k},\mathbf{q}) = \frac{2\pi}{\hbar} |< 0 | \frac{dU}{dz} | k_z > |^2 \left(\frac{\hbar q}{2\rho_0 \omega(q)}\right) \delta(\epsilon_k + \epsilon_0 - \hbar\omega(q) - \epsilon_f).$$

Here n_q, n_k, and N_f are the occupation numbers for ripplons, for atoms in the volume, and for atoms in the adsorbed phase; T is the gas temperature; T_0 is the surface temperature; and m is the mass of the hydrogen atom. The matrix element $\langle 0|dU/dz|k_z\rangle \sim m^{1/4}\epsilon_0^{3/4}k_z^{1/2}$ is evaluated between the wave functions of the free and bound states of the atom in the potential $U(z)$. The wave function of the free state is normalized to a unit flux density.

In determining the Kapitsa jump $\Delta T = T - T_0$ from (6), we must take into account the fact that the surface density of adsorbed atoms (and thus the chemical potential μ_S, which appears in the definition of N_f) in this problem differs from its equilibrium value $n_s(T_0)$, which corresponds to $\Delta T = 0$. An equation relating ΔT to $\Delta n_s = n_s - n_s(T_0)$ is found by equating the fluxes of adsorbing and desorbing particles. The difference between these fluxes, j, is determined by expression (6) without the energy ϵ_k in the integrand.

We first assume that both the volume phase and the adsorbed phase of the atomic hydrogen are classical ideal gases. In this case, for temperatures $T \ll \epsilon_0$, we can set $\epsilon_k = 0$ and $\mathbf{k}_{\parallel} = 0$ in (4) and (5). Carrying out the necessary calculations, using the condition $j = 0$ and dispersion law (3), we find the following expression for the Kapitsa jump from (6) ($\Delta T \ll T_0$):

$$\Delta T = \frac{4\hbar k_*}{5n T_0} \frac{Q}{S} , \qquad (7)$$

where $k_* = C[\hbar\sigma m/(m\epsilon_0)^{3/2}]$, and Q is the heat evolution per unit time. Calculating the matrix element $\langle 0|dU/dz|k_z\rangle$, using a Morse function for $U(z)$ [a Morse function is a good approximation of the actual potential, and at large values of z it becomes the function $-\alpha/z^3(\alpha\simeq 200$ K·Å$^3)$], we find $C\approx 2.5$ and thus $k_* \approx 4\times 10^8$ cm^{-1}.

The way in which the Kapitsa jump increases with decreasing temperature is noteworthy. This behavior is determined entirely by the rate at which atoms are adsorbed on the surface (Refs. 9 and 10; see also the earlier work in Ref. 11).

In calculating the Kapitsa jump by considering only the inelastic scattering of atoms by the surface, we find an expression like (7) with an additional large factor $(\epsilon_0/T_0)^{3/2}$ (cf. Ref. 7).

3. If the surface density of adsorbed atoms is high, we need to incorporate the repulsive interaction of these atoms with each other in the energy conservation law. As a result, ϵ_0 in (4) is replaced by the smaller value $\tilde{\epsilon}_0$. It is important to note, however, that under the condition $\tilde{\epsilon}_0 \gg T$ the adsorption probability is essentially independent of the momentum of the emitted ripplon, which is set by the value of $\tilde{\epsilon}_0$, by virtue of dispersion law (3). On the other hand, momenta f greater than the thermal momentum play an important role in the integrand in (6), causing J to be only a weak function of the degree of degeneracy of the gas of adsorbed particles. Expression (7) therefore remains applicable at all temperatures above the Kosterlitz-Thouless phase transition for the adsorbed gas, T_{cs} (estimates show that at $n\sim 10^{18}$–10^{19} cm^{-3} the temperature T_{cs} lies in the interval 0.1–0.2 K, and $\tilde{\epsilon}_0 \gtrsim 0.5$ K).

4. At $T_0 < T_{cs}$, the elementary excitations in the adsorbed phase are of a collective nature. We take this circumstance into account in the analysis of the adsorption, using the standard procedure of Bogolyubov transformations. It is easy to show that the general expression for the energy flux, (6), remains the same in form, except that $W(\mathbf{k}, \mathbf{q})$ is multiplied by a factor of $mc/2\hbar\kappa$ at $T_0 \ll T_{cs}$, while in energy conservation law (4) (with $\tilde{\epsilon}_0$ in place of ϵ_0) and in the distribution function N_f the energy ϵ_f becomes $\epsilon_f = \hbar cf$, where c is the velocity of sound in the two-dimensional adsorbed gas.

We restrict the discussion to the limiting case $T_0 = 0$. In this case, there is an extreme coverage of the surface by adsorbed atoms, $n_s = n_s(0)$, determined by the condition[12] $\mu_s(T_0 = 0) = \epsilon_0$. (We have digressed from the situation discussed in Ref. 9, which corresponded to a nonequilibrium coverage of the surface.) Under these conditions a particle that has transferred energy to a ripplon as a result of interaction with the surface may end up in a sort of quasistationary state. Although the state that arises should decay by virtue of energy considerations, the one-particle wave function in the direction perpendicular to the surface retains essentially the same localized nature as in the limit $n_s \to 0$ because of the low density of the limiting coverage. The scale time for the decay, accompanied by the emission of a slow atom, of the many-body system that arises turns out to be long. It is easy to see that in this case the transition matrix element from Hamiltonian (1) will have the same value as in a transition to the bound state, so that the preference for energy transfer in comparison with ordinary inelastic scattering remains in force. This circumstance makes this quasiadsorption channel for energy transfer dominant even at $T_0 = 0$, where the renormalized adsorption energy is $\tilde{\epsilon}_0 \approx 0$.

In this case the energy flux J is determined exclusively by the first term in braces (curly brackets) in (6), with $N_f = 0$ and $n_q = 0$. Noting that the energy conservation law becomes

$$\epsilon_k = \hbar\omega(q) + cf , \tag{8}$$

and carrying out some direct calculations, we find the following expression for the Kapitsa jump ($\Delta T = T$):

$$\frac{Q}{S} = \frac{15}{8} A \frac{nT^2}{\hbar k_*} ; \quad T \gg T_c . \tag{9}$$

The coefficient A is a very weak function of T, varying from 1 at $T \gg T_c$ to 0.44 at $T = T_c$, where T_c is the Bose condensation temperature. At $T < T_c$, the coefficient A on the right side of (9) is replaced by the factor 0.44 $(T/T_c)^{3/2}$, which reflects the fact that only particles above the condensate can be adsorbed.

5. Expressions (7) and (9) can be used to evaluate the Kapitsa jump for arbitrary n and T_0. Using the rate constants for volume and surface recombination found earlier[1-5] to determine Q, we easily find that the Kapitsa jump becomes $\sim T_c(n)$ at temperatures well above the Bose condensation point. A result of fundamental importance is that the surface recombination increases with decreasing T_0. Although the energy released at the surface reaches a temperature-independent limiting value as T_0 is reduced further, it follows from (9) that in the limit $T_0 \to 0$ the temperature in the gas is well above $T_c(n)$.

We believe that in order to move into the Bose condensation region it would be necessary to sharply reduce the fraction of the recombination energy that is expended on heating the gas. Such conditions can be realized experimentally by using a single open surface (this situation could be arranged by using a nonuniform magnetic field) and by keeping the dimension of the system in this direction less than or comparable to the mean free path of the particles. In this case the excited molecules and high-energy atoms that are formed in the course of recombination would escape from the system through the open surface before they managed to transfer their excitation energy to the gas particles.

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