Possible induction anomaly of composite materials

A. P. Vinogradov, A. N. Lagar'kov, and A. K. Sarychev
Institute of High Temperatures, Academy of Sciences of the USSR

(Submitted 26 April 1984; resubmitted 13 August 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 7, 296–298 (10 October 1984)

It it shown that composite materials may have, in addition to an inductance governed by the shape of the sample, an internal inductance governed by the geometry of the percolation channels. The internal inductance should become large in the immediate vicinity of the percolation threshold. How the internal inductance affects the electrical properties of composite materials is discussed.

The composite materials in which we are interested here consist of a disordered mixture of a metal and an insulator. The conductivity of such a material vanishes when the metal concentration (by volume) p reaches the percolation threshold p_c. Near p_c, the conductivity of a composite system varies in accordance with $\sigma \sim \tau^t$, and the dielectric constant varies in accordance with $\epsilon \sim |\tau|^{-q}$, where $\tau = (p - p_c)/p_c$, $q = 0.75$ (Refs. 1–3) and $t = 1.8$ (Ref. 4).

The distinctive features in the electrical properties of composite materials at metal

 0021-3640/84/191083-04$01.00

concentrations near p_c stem from the inhomogeneity of the current distribution over the material; specifically, the current flows exclusively along percolation channels, which form an infinite cluster. The scale dimension of the inhomogeneity or the so-called correlation radius ξ, tends toward infinity as the percolation threshold is approached in accordance with $a_0|\tau|^{-\nu}$, where $\nu = 0.9$ (Ref. 4), and a_0 is the microscopic scale dimension of the problem. For samples with a scale dimension A much larger than ξ, the inductance L of the composite sample can be written

$$L = \frac{V}{2I^2} \int \frac{< \mathbf{j}(0)\mathbf{j}(\mathbf{R}) >}{R} d^3R,$$

where $\mathbf{j}(\mathbf{R})$ is the current density at point $\mathbf{R}$, I is the current flowing through the system, the angle brackets mean an average over the volume, and V is the total volume of the system. We are interested in $L_{\rm in}$, the component of the inductance due exclusively to the internal structure of the percolation channels. To find it, we subtract from L the component ($L_{\rm homo}$) of the inductance due to a homogeneous system of the same shape. The component $L_{\rm in}$ can be expressed in terms of the specific internal inductance l, defined as follows:

$$l = \frac{<\mathbf{j}^2>}{<\mathbf{j}>^2} \int \frac{G_i(R)}{R} d^3R; \qquad G_i(R) = \frac{<\mathbf{j}(0)\mathbf{j}(\mathbf{R})> - <\mathbf{j}(\mathbf{R})>^2}{<\mathbf{j}^2(\mathbf{R})>}, \tag{1}$$

where $G_i(R)$ is the spatial correlation function of the currents. For a sample of uniform cross-sectional area S we would have $L_{in} = lA/S$. The specific inductance l is analogous to a resistivity; it is independent of the shape of the sample and may accordingly be regarded as an electrical characteristic of the disordered system.

Clearly, $G_i(R)$ is significantly different from zero only at distances smaller than ξ, falling off rapidly with a further increase in R. Since the only scale dimension of the problem is the correlation radius ξ, it is logical to assume $G_i(R) \sim (R/a_0)^{-(1+\theta)}$ at $a_0 < R < \xi$ and $G_i(R) \sim \exp(-R/\xi)$ at $R > \xi$. It is easy to show that for a binary mixture we would have $\langle \mathbf{j}^2\rangle/\langle \mathbf{j}\rangle^2 \sim \sigma_{\rm eff}^{-1} \sim \tau^{-t}$. Using (1), we then find that near p_c the specific inductance l varies in proportion to $a_0^2\tau^{-m}$, where $m = t + \nu(1-\theta)$ if $\theta < 1$ or $m = t$ if $\theta > 1$. It is clear that $G_i(R)$ should fall off over distance, so that we have $\theta \geqslant -1$. We can finally write

$$t \leqslant m \leqslant t + 2\nu. \tag{2}$$

To further refine the value of the critical index m, we assume, following Refs. 6 and 7, that near the percolation threshold an infinite cluster is a network whose nodes are separated by a scale length on the order of ξ and are connected by single-filament macroscopic bonds of length $\mathscr{L} \sim \tau^{-\zeta}$, where $\zeta = 1.3$ (Refs. 8 and 9). Since $\zeta > \nu$, at $|\tau| \ll 1$, we have $\mathscr{L} \gg \xi$, so that the macroscopic bond is highly twisted and should have a high inductance. The presence of doubled regions—droplets[5]—in the infinite cluster does not affect the estimate of the inductance, since the contribution of a droplet to the inductance is on the order of the size of the droplet, i.e., the shortest path through the droplet. In general, a macroscopic bond consists of turns of all scale dimensions; a turn of size $2h$ is formed from $k = 2^{\zeta/\nu}$ of size h, etc.[5] Hence we can derive a recurrence relation between the inductance of a macroscopic bond of size $2h$ and one of size h: $L_{mb}(2h) = kL_{mb}(h) + f(h)$, where $f(h)$ is the sum of the self-inductance of the turns of size $2h$ and their mutual inductance with the turns of smaller scale dimensions. Setting $f(h) \sim h^{\alpha}$, we find

$$m = \max\{\zeta + \nu, \alpha + \nu\} \geqslant \zeta + \nu = 2.2 > 2\nu = 1.8. \tag{3}$$

Working from (3), we can estimate the contribution of $L_{in} \sim lA/S$ to the total inductance. The component L_{homo} is on the order of A; hence $(L_{in}/L_{homo}) \sim (\xi/A)^2 \tau^{-(m-2\nu)}$. We thus see that L_{in} is comparable to L_{homo} only if $\xi \sim A\tau^{(m-2\nu)/2}$.

We further assume that an external electric field is applied to the system and that this field satisfies the quasistationary condition $\lambda > \xi$, where $\lambda = 2\pi c/\omega$ is the wavelength. At $p < p_c$, the infinite cluster decomposes into finite fragments of ξ. If $\omega < \sigma_M |\tau|^t$, where σ_M is the conductivity of the metal, the current flows primarily along metallic channels, and the breaks in the infinite cluster are bridged by bias currents.[1,2] This picture of the current distribution suggests that the correlation function $G_i(R)$ and, correspondingly, the dependence of l on p are the same below p_c as above p_c and are independent of the frequency. As p_c is approached, the scale dimension ξ of the fragments of the infinite cluster increases, and the relative distance between fragments decreases. As a result, there is an increase in the effective dielectric constant $\hat{\epsilon}_{eff}$. The increase in $\hat{\epsilon}_{eff}$ is accompanied by a simultaneous increase in the inductance of the fragments of the infinite cluster. The constant $\hat{\epsilon}_{eff}$ is found by taking an average over scale lengths on the order of ξ and should be independent of the external geometry. Consequently, it is l that enters $\hat{\epsilon}_{eff}$ while to determine L_{homo}, we need to solve Maxwell's equations into which we substitute $\hat{\epsilon}_{eff}$ with l. Repeating the arguments of Ref. 2, we can derive the estimate

$$\hat{\epsilon}_{eff} = \epsilon'_{eff} + i\epsilon''_{eff} \sim \tau^{-q} \Big/ \left\{ \left[1 - \left(\frac{2\pi a_0}{\lambda} \right)^2 \tau^{-(m+q)} \right] - i \frac{\omega}{\sigma_M} \tau^{-(q+t)} \right\}. \tag{4}$$

If we ignore the inductance of the percolation channels we find that ϵ'_{eff} increases monotonically as $p \to p_c - 0$. Incorporating l results in a qualitative change in the p dependence of ϵ'_{eff}: At $\tau \sim \tau^* = (2\pi a_0/\lambda)^{2/(m+q)}$, we may see a resonance. Near τ^*, the value of ϵ'_{eff} decreases, vanishing, and perhaps even changing sign. We wish to emphasize that these arguments are correct only if the correlation radius ξ is smaller than the wavelength in the medium, $\lambda_{med} = \lambda/\sqrt{\epsilon'_M}$, where ϵ_M is the maximum value of ϵ'_{eff} from (5). The electrical properties of composite materials in the case $\xi > \lambda_{med}$ remain an open question.

We wish to thank B. I. Shklovskiĭ and S. P. Obukhov for a useful discussion of this study.

[1]A. L. Efros and B. I., Shklovskiĭ, Phys. Status Solidi **b76**, 475 (1976).
[2]V. E. Dubrov, M. E. Levinshteĭn, and M. S. Shur, Zh. Eksp. Teor. Fiz. **70**, 2014 (1976) [Sov. Phys. JETP **43**, 1050 (1976)].

[3]A. P. Vinogradov, A. M. Karimov, A. T. Kunavin, A. N. Lagar'kov, A. K. Sarychev, and N. A. Stember, Dokl. Akad. Nauk SSSR **275**, 590 (1984) [Sov. Phys. Dokl. **29**, 214 (1984)].
[4]B. I. Shklovskiĭ and A. L. Éfros, Élektronnye svoĭstva legirovannykh polupovodnikov (Electronic Properties of Doped Semiconductors), Nauka, Moscow, 1979.
[5]A. P. Vinogradov and A. K. Sarychev, Zh. Eksp. Teor. Fiz. **85**, 1144 (1983) [Sov. Phys. JETP **58**, 665 (1983)].
[6]A. S. Skal and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **8**, 1586 (1974).
[7]P. G. de Gennes, J. Phys. (Paris) **37**, L1 (1976).
[8]S. Kirkpatrick, in: AIP Conference Proceedings, Vol. 58, 1979, p. 79.
[9]A. K. Sarychev and A. P. Vinogradov, J. Phys. C **12**, L681 (1979).

Translated by Dave Parsons
Edited by S. J. Amoretty