Cancellation of the plasma-induced shift of the cyclotron resonance in an inhomogeneous electron system

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An explanation is proposed for the absence of a plasma-induced shift of the cyclotron resonance in the Voigt geometry in the experiments by Horst *et al.* and Zhao *et al.* In an inhomogeneous system, the plasma-induced shift is largely cancelled by a renormalization of the cyclotron frequency due to the self-consistent field of the electrons.

Horst *et al.*[1] and Zhao *et al.*[2] have recently reported experiments on the cyclotron resonance of electrons in inversion channels in metal-insulator-semiconductor structures in InSb (Ref. 1) and $Hg_{1-x}Cd_xTe$ (Ref. 2) in the Voigt geometry: a magnetic field (along the x axis) parallel to the surface of the sample and electromagnetic radiation incident normally on the surface (along the z axis), polarized perpendicular to the magnetic field, i.e., along the y axis. We know that in a spatially homogeneous three-dimensional plasma the absorption resonance for the Voigt geometry sets in at the frequency $\omega_{res} = \sqrt{\omega_c^2 + \omega_p^2}$, where ω_c and ω_p are the cyclotron and plasma frequencies, respectively. Both groups of investigators[1,2] found an unexpected and surprising result: the apparent absence of a plasma-induced shift in the resonant transmission of

IR light by the electrons in the inversion channel. We wish to emphasize that the effect involved here is not a weak one: Evaluating ω_p for the typical volume electron density in the channel,[1] we find ω_p to be comparable in magnitude to ω_c. Nevertheless, the resonant peak does not shift when the carrier density in the channel is changed by more than an order of magnitude. In the present letter we propose an explanation for this result that is valid at least for strong magnetic fields, for which the magnetic scale length l is much shorter than the "electrical" scale length L (the typical thickness of the inversion layer). According to our theory, the effect is peculiar to spatially inhomogeneous systems.

Our problem is to derive the electron contribution to the transmission coefficient of a slab of magnetized plasma for an electromagnetic wave. Because of the inhomogeneity of the plasma, the components of the conductivity tensor are integral operators; i.e., they determine a nonlocal relationship between the current and the field. In the Voigt geometry, an extraordinary wave with transverse (E_y) and longitudinal (E_z) field components propagates through the plasma. These field components can be found from Maxwell's equations:

$$\frac{d^2E_y}{dz^2} + \frac{\epsilon_0(z)\,\omega^2}{c^2} E_y = -\frac{4\pi i\omega}{c^2}\hat{\sigma}_{y\alpha}E_\alpha\,, \tag{1}$$

$$\epsilon_0(z)E_z = -\frac{4\pi i}{\omega}\hat{\sigma}_{z\alpha}E_\alpha\,, \quad \alpha = y, z\,.$$

The fields evidently depend on only one coordinate, the z coordinate, so that the second equation contains no derivatives; $\epsilon_0(z)$ is the background dielectric constant of the system (which does not include the electron contribution); and ω is the frequency of the electromagnetic wave.

We now make use of the small parameter l/L. The motion of the electrons along the z axis is described by wave functions with a scale dimension l, and the coordinate of the center of the orbit varies over an interval of the order of L in size. If we retain only the matrix elements of lowest order in l/L in the Kubo equations, the operators $\hat{\sigma}_{\alpha\beta}$ become diagonal (in $\mathbf{r}$ space):

$$\hat{\sigma}_{\alpha\beta}E_\beta \equiv \int dz'\sigma_{\alpha\beta}(z,z')E_\beta(z') \approx \sigma_{\alpha\beta}(z)E_\beta(z)\,, \tag{2}$$

where $\sigma_{\alpha\beta}(z)$ are the components of the local conductivity, which are proportional to the volume electron density $n(z)$. In other words, we obtain a problem of a plasma slab of variable density. The size of the electron orbit is much smaller than the distance over which the electron density changes substantially.

After the variables x and y are separated in the Schrödinger equation, we have an effective potential energy

$$U_{\text{eff}}(z) = \frac{m\omega_c^2}{2}(z - l^2p_y)^2 + V(z)\,. \tag{3}$$

We have chosen the vector potential gauge $\mathbf{A} = (0, -Hz, 0)$; p_y is a conserved momentum component; and $V(z)$ is the complete single-particle potential, which includes the contributions of the external field (i.e., the voltage across the metal-insulator-semiconductor structure), the fields of the ionized impurities, and the self-consistent field of the electrons of the inversion layer. Since the motion along the z axis is finite, and since it is approximately oscillatory in sufficiently strong magnetic fields, we can expand $U_{\rm eff}$ for each electron with a given p_y around the point of the minimum, z_0, and restrict the discussion to quadratic terms. A frequency renormalization then occurs in the Landau oscillator: $\omega_c^2 \to \tilde{\omega}_c^2 \equiv \omega_c^2 + V''(z_0)/m$. In this "parabolic" approximation, the components $\sigma_{\alpha\beta}(z_0)$ can be found easily[3]:

$$\sigma_{yy}(z_0) = \frac{ie^2 n(z_0)}{m^2\,\bar{\omega}}\,\frac{m\bar{\omega}^2 - V''(z_0)}{\bar{\omega}^2 - \tilde{\omega}_c^2(z_0)}\ ; \qquad \sigma_{zz}(z_0) = \frac{ie^2 n(z_0)\bar{\omega}}{m\,(\bar{\omega}^2 - \tilde{\omega}_c^2(z_0))}$$

$$\sigma_{yz}(z_0) = -\,\sigma_{zy}(z_0) = \frac{e^2\omega_c n(z_0)}{m\,(\bar{\omega}^2 - \tilde{\omega}_c(z_0))}\ , \qquad (4)$$

where $\bar{\omega} = \omega + i\nu$, where ν is the phenomenological electron collision rate.

Substituting (2) and (4) into (1), and eliminating E_z, we find an equation for E_y, which can be solved in the approximation of a δ-function barrier (the wavelength of infrared light is three or four orders of magnitude greater than the thickness of the inversion layer). As a result, we find an expression for the amplitude of the transmission coefficient, t, which has additive contributions from electrons with a given z_0:

$$t = \frac{2}{1+\sqrt{\epsilon_0}}\left\{1 + \frac{4\pi i e^2}{(1+\sqrt{\epsilon_0})m\bar{\omega}}\int dz_0 n(z_0)\,\frac{\omega\left[\bar{\omega}^2 - \dfrac{V''(z_0)}{m}\right] - \bar{\omega}\,\omega_p^2(z_0)}{\omega[\bar{\omega}^2 - \tilde{\omega}_c^2(z_0)] - \bar{\omega}\,\omega_p^2(z_0)}\right\}^{-1}. \qquad (5)$$

Corresponding to each group of electrons with a given z_0 is a local plasma shift $\omega_p^2(z_0)$, but the resonant denominator contains, instead of ω_c, the hybrid frequency $\tilde{\omega}_c$. We can thus write

$$\omega_{\rm res}^2 = \omega_c^2 + \frac{V''(z_0)}{m} + \omega_p^2(z_0). \qquad (6)$$

The quantity $V''(z_0)$ is determined in the self-consistent-field approximation by the Poisson equation. It is important to note that a strong external field that curves the bands in a semiconductor does not contribute to $V''(z_0)$, since the corresponding charges lie outside the inversion layer. We can thus write

$$V''(z_0) = -\frac{4\pi e}{\epsilon_0}\,[\,en(z_0) + \rho_{\rm imp}\,], \qquad (7)$$

where e is the electron charge, and $\rho_{\rm imp}$ is the z_0-independent charge density of ionized impurities. In a spatially homogeneous system, we would have $en = -\rho_{\rm imp}$ and $V'' = 0$ by virtue of electrical neutrality, so that there would be a plasma shift of the cyclotron resonance. In this case the term $en(z_0)$ in (7) completely cancels $\omega_p^2(z_0)$ in (6). The shift of the resonance, proportional to $\rho_{\rm imp}$, does not depend on the surface electron density $N_s = \int dz n(z)$. The independence of the position of the resonance peak from N_s is a direct result of the experiments of Refs. 1 and 2. Incidentally, the shift due to ρ_{imp} is far smaller than the expected plasma shift, since the condition $|e|n(z) \gg |\rho_{imp}|$ holds in the inversion channel.

According to the results discussed above, a dependence of ω_{res} on N_s could arise only as a result of anharmonicity, i.e., of the higher derivatives of U_{eff}. On the other hand, the shape of the observed cyclotron-resonance line is determined by the superposition of lines of similar shape, with a common center ω_c and different widths, that depend on $\omega_p^2(z_0)$ [see (5)]. The total width of the cyclotron-resonance line should therefore depend on N_s even in the harmonic approximation. However, there have so far been no reliable experimental studies of this dependence.

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