Electrodynamics of ideal Hall conductors

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Under conditions corresponding to the quantum Hall effect the resistance of a boundary current contact becomes quantized: $R_{qu} = h/2Ne^2$, where N is the number of filled Landau levels. An electric circuit for quantum voltage multiplication is proposed.

Fang and Stiles[1] have recently measured the resistances of various electric circuits containing field-effect transistors operated under conditions corresponding to the quantum Hall effect. Their experimental data can be interpreted by introducing the concept of an ideal $2D$ Hall conductor, for which the $2D$ current density $\mathbf{j}$ and the component of the electric field in the plane of the layer ($z = 0$), $\mathbf{E}_{\|}$, are related by the constitutive equation

$$\mathbf{j} = \sigma_{xx}\mathbf{E}_{\|} - \sigma_{xy}^{(0)}[\mathbf{E}_{\|},\mathbf{k}]; \qquad \mathbf{k} = \mathbf{B}/B \parallel z, \tag{1}$$

where $\epsilon = \sigma_{xx}/\sigma_{xy}^{(0)} \to 0$, and B is the magnetic field. Under the conditions of the quantum Hall effect, when some integer number N of Landau levels is filled in the $2D$ electron system, a typical value of the parameter ϵ is $\epsilon \sim 10^{-5}$ (Ref. 2), since the dissipative conductivity component σ_{xx}, which is of a hopping nature, is anomalously low in the limit[3] $T \to 0$, while the Hall component σ_{xy} is equal to its ideal value $\sigma_{xy}^{(0)} = Ne^2/h$, regardless of whether there are localized electron or hole states in the tails of the Landau levels.[4] We consider only the homogeneous situation, in which precisely the same number (N) of Landau levels is filled everywhere in the sample, and we assume $\sigma_{xy}^{(0)}(N) = \text{const}$.

In the steady state, we have $\mathbf{E} = -\vec{\nabla}\varphi$, and the equation $\operatorname{div}\mathbf{j} = 0$ gives us an equation for determining the potential in the plane of the ideal $2D$ Hall conductor:

$$\Delta_2\varphi(x, y, z = 0) = 0,$$

where $\Delta_2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The boundary conditions on this equation are found from (1) in the limit $\epsilon \to 0$. We assume that on the part (b, c) of the boundary of the conductor (Fig. 1) the normal component of the current density vanishes: $j_n = \mathbf{j}\cdot\mathbf{n} = 0$, where $\mathbf{n}$ is the unit vector of the normal external to the boundary. The derivative of the potential along the boundary is then $d\varphi/dl = \mathbf{n}[\nabla\varphi,\mathbf{k}] = 0$, $\varphi(B) = \varphi(c)$. We now assume that a current I flows across the region (a, b) of the boundary. We circumscribe the contour of the conductor in a clockwise sense ($abcde$ in Fig. 1), assuming that currents flowing into the sample are positive, while currents flowing out are negative. Integrating over the contour $aa'b$, we find the current flowing into the sample to be

$$I = -\int_a^b \mathbf{j}\cdot\mathbf{n}\,dl = \sigma_{xy}^{(0)}\int_a^b \vec{\nabla}\varphi[\mathbf{n},\mathbf{k}]\,dl = \sigma_{xy}^{(0)}[\varphi(b) - \varphi(a)]\,. \tag{2}$$

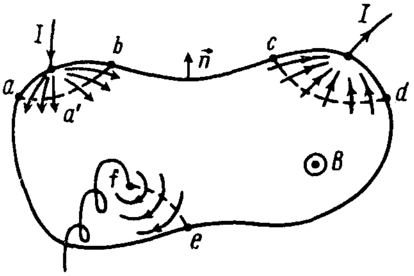

FIG. 1.

The same equation holds for the outgoing current. Consequently, for $I > 0$ the discontinuity in the potential at the boundary of the conductor in the direction we are taking around the contour is positive, $\varphi(b) > \varphi(a)$, while for the outgoing current the corresponding jump is negative. Clearly, the magnitude of the jump does not depend on the positions of the points a and b on the contour of the conductor, as long as the condition $j_n(a) = j_n(b) = 0$ holds. Since the potential is single-valued, the sum of all its jumps along the closed contour is zero, so that the total current across the boundary of the conductor, by virtue of (2), is zero, as expected.

In the limit $\epsilon \to 0$, the dissipation of energy during the current flow is negligible in the conductor, but near the current contact the electric field has a power-law singularity, and there is a finite dissipation. Let us assume that the conductor fills the region $0 \leqslant \theta \leqslant \theta_0$ and abuts an ideal electrode along the ray $\theta = \theta_0$ (Fig. 2). At the boundary with the ideal electrode the potential is constant, $\varphi(r,\theta_0) = \varphi_1$; inside the ideal $2D$ Hall conductor we have $\Delta_2\varphi(r,\theta) = 0$; and on the ray $\theta = 0$ we have $j_n = 0$. The solution of this problem is

$$\varphi = \varphi_1 + Ar^s \sin\{s(\theta_0 - \theta)\}; \operatorname{tang}(s\theta_0) = \epsilon. \tag{3}$$

In the limit $\epsilon \to 0$, we have $s = \epsilon/\theta_0$, and the constant $A = -I/\sigma_{xy}^{(0)}$ is found from the condition that a given current $I > 0$ flows across the contact. Here we have $\varphi(r,0) = \varphi_0 = \varphi_1 - I/\sigma_{xy}^{(0)}$, in agreement with (2). From (3) we find the power dissipat-

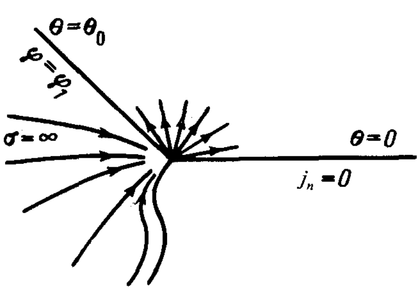

FIG. 2.

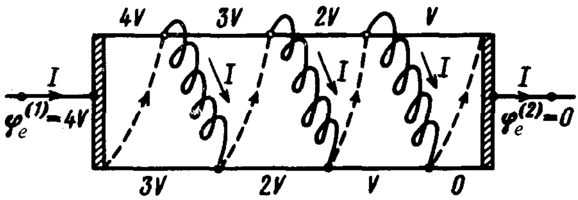

FIG. 3.

ed near the contact to be

$$W = I^2 / 2\sigma_{xy}^{(0)} = I(\varphi_1 - \varphi_0)/2 = I^2 R_{qu}, \tag{4}$$

where $R_{qu} = h/2Ne^2$ is the quantized resistance of the contact. Clearly, because of the singularity at $r = 0$, this resistance is insensitive to the geometric shape of the contact. It is important to note, however, that for an arbitrary current direction a singularity arises at the right-hand corner of the ideal electrode (the first corner encountered as the contour is circumscribed; Fig. 2). Accordingly, the potential of this electrode, φ_e, is always the same as the potential of the part of the boundary of the Hall conductor that lies beyond the discontinuity.

These rules, along with the standard Kirchhoff laws and the fact that the potential of any connection of an ideal electrode is constant, are sufficient for calculations for circuits with ideal $2D$ Hall conductors. For the example in Fig. 3, we have designed a circuit that can be used for quantum voltage multiplication. The standard jump is $V = I/\sigma_{xy}^{(0)} = IR_H$. The resistance of the circuit in Fig. 3 is $R = [\varphi_e^{(1)} - \varphi_e^{(2)}]/I = 4R_H$, corresponding to the presence of eight contacts $R_{qu} = R_H/2$ connected in series (the dashed curves show the current path in the Hall conductor). The accuracy of the voltage multiplication is determined by the small parameter ϵ and by the resistance of the ideal electrodes, which can apparently be made arbitrarily low (by using a superconductor).

In addition to the boundary contacts discussed here, volume contacts are conceivable in principle; such contacts could be circumscribed along a contour that lies entirely inside the Hall conductor (contact f in Fig. 1). In contrast with a boundary contact, the resistance of a volume contact would be $R_{vol} \sim 1/\sigma_{xx}$, and in the limit $\epsilon \to 0$ such contacts would actually be voltage sources. Around each volume contact there are closed Hall currents; the total current crossing, for example, the contour (f,e) in Fig. 1 is $I = \sigma_{xy}^{(0)} |\varphi(f) - \varphi(e)|$.

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[2]K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[3]N. A. Usov, F. R. Ulinich, and Yu. B. Grebenschikov, Solid State Commun. **43**, 475 (1982).
[4]N. A. Usov and F. R. Ulinich, Zh. Eksp. Teor. Fiz. **86**, 644 (1984) [Sov. Phys. JETP **59**, 376 (1984)].

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