Oscillations of a Bloch line in a domain wall

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A theory of the oscillations of a Bloch line in a domain wall is constructed on the basis of an equation of motion that includes the gyrotropic force. This force determines the unique properties of these oscillations, in particular, the effective mass of the Bloch line.

Bloch lines (BL) are topologically stable linear defects of the magnetization field **M** that separate subdomains of the domain wall (DW). These lines play an important role in the dynamics of magnetic bubbles, used to store information in computers.[1] Experiments have now begun on the use of the BL for storing information.[2] This circumstance has made it extremely important to study the motion of BL along DW, to which adequate attention has not been devoted in the past. The first experiments along these lines revealed a new and interesting phenomenon: resonant oscillations (free and forced) of BL.[3,4] The mass of the Bloch line determined from the frequency of this resonance turned out to be several orders of magnitude greater than the mass found by Ignatchenko and Kim.[5] Their paper was the first theoretical study in which the oscillations of a BL in a DW were examined.

In this letter we expound a theory of linear oscillations of an isolated BL based on an equation of motion that includes the gyrotropic force which was ignored by Ignatchenko and Kim. The theory explains the observed properties of the resonance oscillations.

We examine a Bloch line parallel to the Z axis in a DW situated in the XZ plane. The displacement of the DW far from the BL, whose position is given by the two-dimensional vector $\mathbf{r}(x,y)$ is denoted by y_0 (see Fig. 1). The quantity $(y - y_0)$ determines the depression in the DW. The following equation[1,6] can be derived for the vector $\mathbf{r}$ (there are no magnetic fields oriented along the y axis):

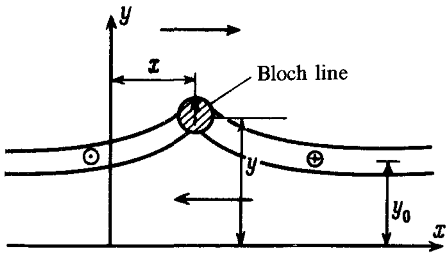

FIG. 1. A Bloch line in a domain wall. The arrows mark the orientation of the moment **M** in the domains and subdomains (out of the page—the moment **M** in the subdomain to the left of the BL). The arrow in the BL indicates the orientation of the magnetization in it. In this case, $\nu = 1$.

$$2\pi \frac{M}{\gamma} \nu \, [\hat{\mathbf{z}} \times \dot{\mathbf{r}}] = - \frac{\partial F}{\partial \mathbf{r}} , \tag{1}$$

where γ is the gyromagnetic ratio, and $\nu = \pm 1$ is the topological charge determined by the direction of rotation of the vector $\mathbf{M}$ at the center of the BL and by the change of the projection M_z in it. We can write the effective free energy $F(\mathbf{r})$ in the form

$$F(\mathbf{r}) = \frac{\lambda}{2} x^2 + \frac{B}{2} (y - y_0)^2 + 2\Delta_0 M H_z x , \tag{2}$$

where Δ_0 is the effective thickness of the DW. The first term in (2) determines the x component of the magnetostatic restoring force, whose parameter λ depends on the specific geometry and is determined experimentally by measuring the displacement induced by a constant field H_z (along the z axis) that interacts with the moments of the subdomains and is thus the field that propels the BL [the third term in (2)]. The second term in (2) is the y component of the restoring force, which is governed by the finite rigidity of the DW. For simplicity, we dropped the relaxation term in Eq. (1).

According to (1), the static force acting on the BL is equal to the gyrotropic force,[1)] which is proportional to the velocity and which is analogous to Magnus' force for vortices. This static force is not equal to the inertial force, which is proportional to the acceleration $\ddot{r}$, as in Newton's second law. In addition to the gyrotropic force, the inertial force, which appears in Ignatchenko and Kim's calculations,[5] could also have been included on the left side of (1), but in the case at hand this would exceed the accuracy of the calculations.

Eliminating the coordinate y from the two first-order equations for the components x and y of the vector $\mathbf{r}$ [the vector equation (1)], we obtain a single second-order equation for x

$$m_L \ddot{x} + \lambda x = - 2\Delta_0 M H_z + 2\pi \nu \dot{y}_0 \frac{M}{\gamma} . \tag{3}$$

We thus see that although the inertial force is ignored in the original equation (1), the inertial term with the mass of the BL

$$m_L = \frac{1}{B} \left(\frac{2\pi M}{\gamma} \right)^2 \tag{4}$$

appears in the equation for the displacement x of the BL along the DW. This mass, however, is purely gyrotropic in origin and may be attributable to the pressure of a depression in the DW and the motion of the BL across it. The gyrotropic nature of the law governing the motion of the BL is manifested in the fact that the oscillations of the BL are always elliptically polarized and can be excited not only by the field H_z, but also by the field H_x, which displaces the DW ($\dot{y}_0 \neq 0$), without acting directly on the BL. Experiments[4] have confirmed that the field H_x can induce oscillations of the BL (it is even more effective than the field H_z) and that these oscillations are elliptically polarized, as manifested by the fact that the resonance is deduced simultaneously from the oscillations of the BL and from the oscillations of the DW.

The elliptically polarized oscillations of the BL in the potential well created by the magnetostatic forces are to a large extent analogous to the circularly polarized Thompson oscillations of a vortex in hydrodynamics. On the other hand, an oscillation of the BL is at the same time an oscillation of the field of moment **M**, i.e., magnons. Flexural oscillations of the DW are magnons trapped on the surface of the DW, i.e., surface magnons, whereas the oscillations examined by us are magnons trapped on the BL.

By assuming that the shape of the DW, which is curved due to the motion of the BL, is determined by the surface tension of the DW and by the magnetostatic restoring force that acts on the DW and holds it in a definite position, we find that $B = 2\lambda_0/q$, where λ_0 is the restoring-force parameter equal to the ratio of the displacement of the DW to the field H_x causing its static displacment, while the wave number $q = \sqrt{\lambda_0/\sigma}$ determines the distance $1/q$ over which the BL bends the wall. Here σ is the energy density of the wall. If in determining σ the magnetic charge that appears as a result of the bending of the wall is ignored, i.e., if the effect of magnetostatics on the flexural rigidity is ignored (see Sec. 22A in Ref. 1), then we find that $B = 300$ ergs/cm^3 and $q = 0.3 \times 10^3$ cm^{-1}, after determining λ_0 from Fig. 5 in Ref. 4. In the geometry of the experiment in Ref. 4, however, the magnetostatics increases the flexural rigidity of the DW and there apparently arises a situation where the distance $1/q$ over which the BL bends the DW is larger than the length L between the BL, which in the experiment is on the order of 10^{-2} cm. In this case, the BL oscillate, entraining the DW in step, essentially without bending it.[2)] As an estimate of B, in this case we must take the value $B = \lambda_0 L \approx 500$ ergs/cm^3. The values of B calculated from Eq. (4) in terms of the mass m_L determined from the experimentally measured resonances are largely approximately equal to our theoretical estimate of B in terms of the rigidity of the DW.

The theory of oscillations of a BL formulated above assumes that the BL is much shorter than the length of the section of DW in which the bending occurs. For this reason, the main losses due to dissipation must originate in this section, rather than within the BL. This is important for determining the mobility of the BL along the DW.

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[1)]This force does not depend on the precise shape of the field **M** within the BL [see the derivation of Eq. (12.60) in Ref. 1].

[2)]In this connection, see Ref. 7, where oscillations of magnetic bubbles with a BL are examined, without allowance for the bending of the DW.

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