# Possible solution of the initial-perturbation problem in cosmology

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A new model is proposed in which the amplitude required of the initial perturbations arises in a natural way.

Theories with inflation are presently the subject of active research.[1–3] Inflation is required to solve several cosmological problems, among them causal connectedness, planarity, the monopole problem, homogeneity, and isotropy. During the decay of the inflation stage, the quantum fluctuations of the scalar field grow and give rise to the inhomogeneities in the universe which are responsible for the formation of galaxies. This property is a major improvement of the inflation scenario, since previously the initial-perturbation spectrum had to be worked into the theory "manually."[4] However, the perturbation amplitude in the ordinary grand unified theories is too large to satisfy the observed isotropy of the background radiation.[5–7] Other versions have also been proposed.[8–13] All the models proposed in Refs. 8–12 were criticized in Refs. 13 and 14. Here we will mention only the major features of these models, which make it possible to achieve the necessary perturbation amplitude.

To find the amplitude and spectrum of the perturbations, we need to study the dynamics of a one-component scalar field $\varphi$ with a self-action $\lambda\varphi^4$ or $\tilde{\lambda}\tilde{\mu}\varphi^3$. In order to obtain the required perturbation amplitude, we require that the dimensionless constants $\lambda$ and $\tilde{\lambda}$ (or $\tilde{\mu}/M_{Pl}$, where $M_{Pl}$ is Planck's mass) be extremely small, e.g., $\lambda \lesssim 10^{-12}$. In general, the introduction of such small dimensionless constants in the theory requires a special explanation.

One possibility is related to the supersymmetry theories,[8,9] in which the effective coupling constant $\lambda(\varphi)$ is small because the contributions of bosons and fermions cancel out. The typical perturbations that arise during the phase transition, however, are too small.[8] To obtain the necessary perturbations, we need to incorporate in the theory some very small constants, on the order of $10^{-8}$–$10^{-9}$, but this is an unnatural approach; furthermore, there is the problem of heating after the phase transition.[13,14]

In Refs. 10 and 11 the coupling constant $\lambda$ is the square of some other dimensionless constant that is initially introduced: $\lambda_1 \sim 10^{-6}$, again exceedingly small.

In the models based on an $N=1$ supergravity which is interacting with matter,[12,13] the interaction constants satisfy $\lambda, \tilde{\lambda} \propto (\mu/M_{Pl})^6$, where $\mu$ is a constant that has the dimension of mass. The interaction constants $\lambda$ and $\tilde{\lambda}$ are small in this case because $\mu$ is small in comparison with Planck's mass $M_{Pl}$. The physical meaning of $\mu$ is unclear, however, and we do not know what a natural value of $\mu$ would be.

This model is based on the circumstance that there is an interesting result in the grand unified theories (GUTs). The gauge constants of the strong, weak, and electromagnetic interactions are equal at an energy $M_X$ much smaller than Planck's energy. A natural small parameter thus arises in the GUTs: the ratio of the mass of the superheavy gauge boson to Planck's mass, $M_X/M_{Pl}$, which would be on the order of $10^{-4}$ for the minimal SU(5) theory. In the models of Refs. 5–13 the perturbation amplitude is essentially independent of $M_X$. In the model that we are proposing here, a nonzero expectation value of the scalar field which causes the inflation also gives us Planck's mass; i.e., instead of the Einstein term in the Lagrangian we have $-(-g)^{1/2}R\varphi^2$. The perturbation amplitudes are small, of the magnitude required, because of the small value of $M_X/M_{Pl}$, and it is not necessary to introduce any other small parameters. Consequently, small perturbation amplitudes follow in a *natural* way from field-theory considerations in this model, rather than from a fit to astronomical observations.

Let us examine a theory that is gauge-invariant at the tree level. As in the Coleman-Weinberg model, gauge invariance is broken only by radiative corrections. The theory that we are proposing here is thus a generalization of the Coleman-Weinberg model to the theory of gravitation.

As a model we consider a version of the grand unified theory constructed on the SU(5) group with a singlet $\varphi$. The singlet $\varphi$ interacts with the scalar and spinor fields, and the interaction constants are dimensionless, as follows from the requirement of gauge invariance. For example, the interaction with the Higgs 24-plet $\widetilde{\phi}$ is

$$a(\operatorname{tr}\widetilde{\phi}^2)^2 + b\operatorname{tr}\widetilde{\phi}^4 - \lambda_1\varphi^2\operatorname{tr}\widetilde{\phi}^2. \tag{1}$$

We also require the discrete symmetry $\varphi\to-\varphi$, $\widetilde{\phi}\to-\widetilde{\phi}$. We assume $a, b\gg\alpha^2$, where $\alpha=g^2/4\pi\sim 1/50$ is the gauge coupling constant, and we assume that the radiative corrections in the $\widetilde{\phi}$ sector can be ignored. Furthermore, we make the usual assumptions $a, b\ll\alpha$.

The basic idea of this model is that the energy of the vacuum in this model comes primarily from the fluctuations of vector fields: $V\sim g^4\phi^4\sim M_X^4(\phi/\phi_0)^4$, where $\phi$ is the principal component of the 24-plet $\widetilde{\phi}$, where $\widetilde{\phi}=\phi\,\mathrm{diag}(1,1,1,-3/2,\ -3/2)$, and $\phi_0$ is the value of $\phi$ at equilibrium. As a consequence of gauge invariance in the $\widetilde{\phi}$ sector, we find $\phi\propto\varphi$ and $\phi/\phi_0=\varphi/\varphi_0$, where $\varphi_0$ is the value of $\varphi$ at equilibrium. If the field $\varphi$ is normalized in such a manner that the coefficient of the kinetic term for $\varphi$ becomes unity, then $\varphi_0$ becomes on the order of $M_{Pl}$. We thus have $V\sim(M_X/M_P)^4\varphi^4$, and the constant of the quaternary interaction, $\lambda$, turns out to be small in a natural way, as a consequence of the small value $M_X/M_{Pl}\sim10^{-3}$. In addition, the dimensionless perturbation amplitude turns out to be small, proportional to[5–7] $\lambda^{1/2}$.

The Lagrangian of our theory is

$$\mathcal{L} = [-\varphi^2R - V(\varphi) + \frac{\omega}{2}(D_\mu\varphi)^2](-g)^{1/2} + \mathcal{L}_{\mathrm{rem}}, \tag{2}$$

where $\omega$ is a dimensionless constant, $\mathscr{L}_{\mathrm{rem}}$ is the remaining part of the Lagrangian, which includes other fields, different from $\varphi$, and the interaction of these fields with $\varphi$. At the tree level we have $V(\varphi)=\lambda_0\varphi^4$, but the incorporation of fluctuations of the vector fields leads to the potential

$$V_{eff}(\varphi) = \frac{1}{4}\beta\varphi^4\left(\ln\frac{\varphi}{\varphi_0} - \frac{1}{4}\right) + \frac{\beta}{16}\varphi_0^4, \tag{3}$$

where $\beta=1152(M_X/M_{Pl})^4$, and $M_X$ is the mass of a superheavy gauge boson. The last term in (3) is added to satisfy the condition $V_{\mathrm{eff}}(\varphi_0)=0$, which means that there is no cosmological term in the present epoch.

The evolution begins at strong fields, $\varphi_{\mathrm{in}}\gg\varphi_0$. It can be shown that the universe expands in a quasiexponential fashion until $\varphi$ becomes comparable to $\varphi_0$. In order to solve the familiar cosmological problems cited in Ref. 1, we require $\varphi_{\mathrm{in}}>300\varphi_0$. This scenario has much in common with Linde's random scenario.[13]

It can be shown that the zero-point fluctuations of the scalar field in theory (2) give rise to perturbations of the metric with a mean square value[15]

$$h_k = 4\,(12)^{1/4}(1+\omega/12)^{-1/4}\frac{M_X^2}{M_{Pl}^2}k^{-3/2}\ln^{3/4}(k_1/k). \tag{4}$$

The notation is that of Refs. 6 and 16. Expression (4) holds for $\ln(k_1/k)\gg 1$. Starobinskiĭ[16] has derived limitations directly on the quantity $k^{3/2}h_k$, at the level of $10^{28}$ cm [$\ln(k_1/k)\simeq 70$]. It follows from observations of the anisotropy of the background radiation that we have $k^{3/2}h_k < 1.5\times 10^{-3}$. If the perturbations are to be able to form galaxies, we must require[16,17] $k^{3/2}h_k > 1.5\times 10^{-4}$. [The probable value is[16] $k^{3/2}h_k = (0.3-1)\times 10^{-3}$.] In our case, this value is reached at $M_X = (1-3)\times 10^{16}$ GeV (if we assume $\omega \lesssim 10$)—an extremely plausible value.

We note that $\lambda_1$, a small value, is proportional to $(M_X/M_{Pl})^2$, but the important point is that the small value of $\lambda_1$ in our model is not an obvious contrivance but instead a consequence of the unification of the strong, weak, and electromagnetic interactions.

The singlet $\varphi$ can also be used to solve the problem of the strong CP noninvariance, through the mechanism of an invisible axion.[18] In this case $\varphi$ is a complex field.

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[1]A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2]A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3]A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[4]Ya. B. Zel'dovich and I. D. Novikov, Stroenie i évolyutsiya Vselennoĭ (Structure and Evolution of the Universe), Nauka, Moscow, 1975.
[5]S. W. Hawking, Phys. Lett. **115B**, 295 (1982).
[6]A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[7]A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).
[8]C. E. Vayonakis, Phys. Lett. **123B**, 39 (1983).
[9]A. Albrecht, S. Dimopoulos, W. Fischler, E. Kolb, S. Raby, and P. J. Steinhardt, Nucl. Phys. **B229**, 528 (1983).
[10]Q. Shafi and A. Vilenkin, Phys. Rev. Lett. **52**, 691 (1984).
[11]J. Ellis, D. V. Nanopoulos, K. A. Olive, and K. Tamvakis, Phys. Lett. **120B**, 331 (1983).
[12]D. V. Nanopoulos, K. A. Olive, M. Srednicki, and K. Tamvakis, Phys. Lett. **123B**, 41 (1983).
[13]A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 606 (1983) [JETP Lett. **37**, 724 (1983)]; Rockefeller University Preprint, 1983; Phys. Lett. **129B**, 177 (1983).
[14]B. A. Ovrut and P. J. Steinhardt, Phys. Lett. **133B**, 161 (1983).
[15]B. L. Spokoĭnyĭ, Preprint No. 8, L. D. Landau Institute of Theoretical Physics, 1984; Phys. Lett. B, 1984, to be published.
[16]A. A. Starobinskiĭ, Pis'ma Astron. Zh. **9**, 579 (1983) [Sov. Astron. Lett. **9**, 302 (1983)].
[17]P. J. E. Peebles, Astrophys. J. **263**, L1 (1982).
[18]S. Y. Pi, Phys. Rev. Lett. **52**, 1725 (1984).

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