Supersymmetry vacuum configurations in a $d = 11$ supergravity

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Some new supersymmetry vacuum configurations of a $d = 11$ supergravity, which are invariant under the group SO(3,2)×SU(3)×SU(2), are derived. The seven-dimensional compact space corresponding to these configurations is an SU(2) instanton on CP^2.

1. Considerable interest has recently been attracted to an analysis of all possible vacuum states of a $d = 11$ supergravity which result from a spontaneous Freund-Rubin compactification[1] of the 11-dimensional space into the direct product of a four-dimensional anti-de Sitter space and a seven-dimensional compact space of the Einstein type, for which the Ricci tensor is

$$R^l_n = -6m^2\delta^l_n \tag{1}$$

(l and n are the world indices of the seven-dimensional space, and m is an arbitrary parameter that has the dimension of reciprocal length). The vacuum configurations of most interest are those with the topology of a seven-dimensional sphere, S^7. In the case of a compactification into a standard S^7 with the symmetry group SO(8), the corresponding vacuum configuration retains the $N = 8$ supersymmetry of the effective four-dimensional sector of the theory, which is reduced to $N = 1$ upon a deformation of the standard S^7 into a so-called squashed sphere with the symmetry group SO(5)×SU(2) (see Ref. 2 and the references there). A study has also been made of the class of homogeneous seven-dimensional Einstein spaces with the symmetry group[3,4] SU(3)×SU(2)×U(1), which allow the existence of vacuum configurations with an $N = 2$ supersymmetry.[4]

2. Witten[3] was the first to call attention to the possibility of establishing a direct relationship between the structure of compactified vacuum configurations in a $d = 11$ supergravity and the gauge theory of strong and electroweak interactions. In order to pursue the realistic direction pointed out by Witten, especially to resolve the question of a joint spontaneous breaking of the supersymmetry group and of the SU(2)×U(1) group of the electroweak interactions, it is necessary to study the allowed vacuum states with the maximum number of unbroken supersymmetries.

In the present letter we are interested in vacuum states which allow $N = 3$ and $N = 1$ supersymmetries in an effective $d = 4$ theory and which are invariant under transformations of the group SO(3,2)×SU(3)×SU(2) [SO(3,2) is the group of motions of AdS^4]. To find vacuum configurations of this type we make use of an analogy: S^7 may be thought of[2] as an SU(2) instanton on an S^4 sphere; i.e., it is an associated stratified space $E(S^4, S^3, \mathrm{SU}(2))$ with a base S^4, a layer S^3 and a structure group SU(2).

The metric of this space has the same form as the metric of the space-time in multidimensional Kaluza-Klein theories[5]:

$$g_{mn} = \begin{pmatrix} g_{ab}(y) + \frac{1}{k_3}\theta^A_a \theta^B_b K^i_A K_{Bi} & \frac{1}{k_3}\theta^A_a(y) K_{Ak}(z) \\ \frac{1}{k_3}\theta^A_b(y) K_{Ai}(z) & g_{ik}(z) \end{pmatrix}, \tag{2}$$

where $g_{ab}(y)$ and $g_{ik}(z)$ are the metrics of S^4 and S^3 with the coordinates y^a and z^i, respectively; $K_{Ai}(z)$ are the Killing vectors on S^3 which correspond to one of the invariant subgroups of the SO(4) group and which are normalized by the condition $K_{Ai}K^A_k = (1/2)g_{ik}(z)$ [A is the SU(2) index]; and k_3 is the average curvature of the S^3, $\theta^A_a(y)$ field of a BPTS instanton on S^4.

We replace S^4 by the projective space $CP^2 = \mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1)$, and we consider the space $E(CP^2, S^3, \mathrm{SU}(2))$, which, as will be shown below, is an SU(2) instanton on CP^2. The space $E(CP^2, S^3, \mathrm{SU}(2))$ is locally isomorphic to $CP^2\times S^3$, has a symmetry group $\mathrm{SU}(3)\times\mathrm{SU}(2)$, and is topologically equivalent to the homogeneous space SU(3)/U(1). We assume that the metric in (2) is the CP^2 metric and that $\theta^A_a(y)$ is a gauge field on CP^2. As was shown in Ref. 6, the stress tensor of the gauge fields on the symmetry spaces G/H is constructed in an orthogonal basis from the structure constants of the group G, one of whose indices corresponds to the holonomic group H or its invariant subgroup. In the case under consideration the tensor $F^A_{ab} = \partial_a\theta^A_b - \partial_b\theta^A_a + C_{BC}{}^A\theta^B_a\theta^C_b$ [$C_{BC}{}^A$ are the structure constants of the SU(2) group] is constructed from the structure constants $f_{(a)(b)}$ of the SU(3) group in a Gell-Mann basis. In an orthogonal basis, this tensor would be written

$$F_{(a)(b)}{}^A = -k_4 f_{(a)(b)}{}^A, \tag{3}$$

where k_4 is the average curvature of CP^2, and the indices (a) and (b) correspond to the space tangent to CP^2. It is easy to show that the structure constants $f^A{}_{(a)(b)}$ satisfy the condition of self-duality, so that the gauge field specified by stress tensor (3) is an SU(2) instanton on CP^2 and is invariant [to within gauge SU(2) transformations] under the SU(3) group.

3. Let us examine the Riemannian structure of the space $E(CP^2, S^3, \mathrm{SU}(2))$. The spin connection $\omega_{(m)(n)p}$ [(m) and (n) are indices of the tangent space] matched to metric (2) is[7]

$$\omega_{(i)(j)k} = \omega^{S^3}_{(i)(j)k}, \quad \omega_{(i)(j)a} = -K^B{}_{(i)}K^C{}_{(j)}C_{BC}{}^A\theta_{Aa},$$

$$\omega_{(i)(a)k} = 0, \quad \omega_{(i)(a)b} = \frac{1}{2\sqrt{k_3}}K_{A(i)}F^A_{(a)b},$$

$$\omega_{(a)(b)i} = -\frac{1}{2k_3}F^A_{(a)(b)}K_{Ai}, \quad \omega_{(a)(b)c} = \omega^{CP^2}_{(a)(b)c} - \frac{1}{4k_3}\theta_{cA}F^A_{(a)(b)}, \tag{4}$$

and the curvature tensor $R^{(m)}_{(n)(l)(p)}$ of connection (4) can be written as follows in the notation of differential form ($\Omega^{(m)}{}_{(n)} = R^{(m)}{}_{(n)(l)(p)} e^{(l)} \lambda e^{(p)}$), with the help of (1) and (3):

$$\Omega^{(a)}{}_{(b)} = \Omega^{CP^2\,(a)}{}_{(b)} - 6m^2 e^{(a)} \lambda e_{(b)},$$
$$\Omega^{(i)}{}_{(k)} = 2m^2 e^{(i)} \lambda e_{(k)}, \quad \Omega^{(i)}{}_{(a)} = 2m^2 e^{(i)} \lambda e_{(a)}. \tag{5}$$

Here, by virtue of (1), the curvatures k_4 and k_3 are expressed in terms of the parameter m by $k_4 = 6k_3 = 24m^2$; $e^{(a)} = e^{(a)}_a(y)dy^a$ and $e^{(i)} = e^{(i)}_i(z)dz^i + \frac{1}{\sqrt{k_3}} \theta^A_a K^{(i)}_A dy^a$ are reference forms on the space $E(CP^2, S^3, \mathrm{SU}(2))$; and

$$K^{(i)}_A = \frac{1}{\sqrt{k_3}} e^{(i)}_i K^i_A \;; \quad \Omega^{CP^2\,(a)}{}_{(b)} = k_4 \left(f^{(a)}{}_{(b)A} f_{(c)(d)}{}^A + f^{(a)}{}_{(b)8} f_{(c)(d)}{}^8 \right) e^{(c)} \lambda e^{(d)} .$$

4. The space $E(CP^2, S^3, \mathrm{SU}(2))$ with the Riemannian structure specified by (1)–(5) is therefore a Freund-Rubin solution. As has been shown elsewhere,[2,4] the maximum number of unbroken supersymmetries of the four-dimensional sector of the theory in Freund-Rubin solutions is characterized by the number of independent eight-component spinors η, which are invariant under transformations $C_{(m)(n)}$,

$$C_{(m)(n)}\eta = (R_{(m)(n)}{}^{(p)(s)} - m^2 \delta^{(p)}_{[(m)} \delta^{(s)}_{(n)]}) \Gamma_{(p)} \Gamma_{(s)} \eta = 0, \tag{6}$$

which generate a subgroup of the Spin (7) group ($\Gamma_{(m)}$ are 8×8 Dirac matrices). In the case under consideration, the $C_{(m)(n)}$ become

$$C_{(i)(j)} = C_{(i)(b)} = 0, \; C_{(a)(b)} = R^{CP^2}_{(a)(b)}{}^{(c)(d)} \Gamma_{(c)} \Gamma_{(d)} - 8m^2 \Gamma_{(a)} \Gamma_{(b)} \tag{7}$$

and, as is easily shown, form a Lie algebra of the SU(2) group which leaves four spinors invariant. It can be shown, however, that the equation $(D_n - (1/2)m\Gamma_n)\eta = 0$, for which (6) is an integrability condition,[2] has only three solutions, so that the vacuum configuration under consideration here preserves the $N = 3$ supersymmetry in the effective $d = 4$ theory.

5. It can be shown that if $k_4 = \frac{6}{5} k_3 = \frac{40}{3} m^2$, then metric (2) is also Einsteinian, and in the case of $E(S^4, S^3, \mathrm{SU}(2))$ it corresponds to a squashed[2] S^7; in the case $E(CP^2, S^3, \mathrm{SU}(2))$ it corresponds to an SU(3)/U(1) space with a deformed canonical metric. Such vacuum configurations preserve the $N = 1$ supersymmetry in $d = 4$.

6. By virtue of the theorem of Ref. 4, the complete symmetry groups of the vacuum states found here are the supergroups $O\mathrm{Sp}(4,3) \times \mathrm{SU}(3)$ and $O\mathrm{Sp}(4,1) \times \mathrm{SU}(3) \times \mathrm{SU}(2)$, respectively.

We note in conclusion that all the known Freund-Rubin solutions and also several new solutions (some of which preserve the $N = 2$ supersymmetry) form a class of spaces in which compactification occurs in two steps. In the first step, as in the classical Kaluza-Klein theory, an S^1 sphere and an Abelian gauge field form. In the second step, this Abelian field causes a compactification of the six-dimensional space and leads to the formation of various vacuum states of a $d = 11$ supergravity.

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