Renormalization in gauge theories with γ_5 anomalies

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A method is proposed for a gauge-invariant renormalization for theories with γ_5 anomalies. The method involves the introduction of a nonlocal gauge-noninvariant counterterm in the Lagrangian. The method is illustrated in the case of an exactly solvable model, a modification of the Schwinger model.

The existence of γ_5 anomalies[1] in gauge theories is known to violate the Ward identities, i.e., to disrupt gauge invariance at the quantum level. It is thus generally assumed that theories with γ_5 anomalies are internally inconsistent. Is there any way to restore gauge invariance in theories with γ_5 anomalies? A trivial way to do this is to introduce in the theory some additional fermions in such a manner that the γ_5 anomalies cancel out in the sum.

In the present letter we wish to propose another way to combat the γ_5 anomalies. The idea is to introduce in the Lagrangian a nonlocal gauge-noninvariant counterterm that restores gauge invariance at the quantum level.

We consider the two-dimensional Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + A_{\mu}(e_V\bar{\psi}\gamma^{\mu}\psi + e_A\bar{\psi}\gamma^{\mu}\gamma_5\psi). \tag{1}$$

With $e_A = 0$, Lagrangian (1) becomes the Lagrangian of the Schwinger model,[2] whose solution is well known. If $e_A \neq 0$, the model has γ_5 anomalies. An integration over the Fermi fields gives us

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e_V^2}{2\pi}A_{\mu}\left(g^{\mu\nu} - \frac{\partial^{\mu}\partial^{\nu}}{\Box}\right)A_{\nu} + \frac{e_A^2}{2\pi}A_{\mu}$$

$$\times\left(-\frac{\partial^{\mu}\partial^{\nu}}{\Box}\right)A_{\nu} - \frac{e_V e_A}{2\pi}A_{\mu}(\partial^{\mu}\epsilon^{\nu\beta}\partial_{\beta} + \partial^{\nu}\epsilon^{\mu\beta}\partial_{\beta})\frac{1}{\Box}A_{\nu}, \tag{2}$$

$$\int\mathcal{L}_{eff}\,d^2x = \frac{1}{i}\ln\int e^{iS}d\bar{\psi}\,d\psi. \tag{3}$$

Expression (2) for $\mathscr{L}_{\text{eff}}$ with $e_A \neq 0$ is not gauge-invariant. To restore gauge invariance to the theory we introduce in Lagrangian (1) a counterterm

$$\Delta\mathcal{L} = \frac{e_A^2}{2\pi}A_{\mu}\left(\frac{\partial^{\mu}\partial^{\nu}}{\Box}\right)A_{\nu} + \frac{e_V e_A}{2\pi}A_{\mu}(\partial^{\mu}\epsilon^{\nu\beta}\partial_{\beta} + \partial^{\nu}\epsilon^{\mu\beta}\partial_{\beta})\frac{1}{\Box}A_{\nu}. \tag{4}$$

Counterterm (4) makes the effective Lagrangian gauge-invariant:

$$\mathcal{L}'_{eff} = \mathcal{L}_{eff} + \Delta\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e_V^2}{2\pi}A_\mu\left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\Box}\right)A_\nu \; . \tag{5}$$

Effective Lagrangian (5) describes a free scalar particle of mass $m = e_V/\sqrt{\pi}$. In the bosonization representation,[3] in which we have

$$i\bar{\psi}\hat{\partial}\psi \sim \frac{1}{2}(\partial_\mu\sigma)^2 , \qquad \bar{\psi}\gamma^\mu\psi \sim \frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\sigma ,$$

$$\mathcal{L} + \Delta\mathcal{L} = \frac{1}{2}\partial_\mu\sigma'\partial^\mu\sigma' - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e_V}{\sqrt{\pi}}\sigma'\epsilon^{\mu\nu}\partial_\mu A_\nu \quad ,$$

$$\sigma' = \sigma + \frac{e_A}{\sqrt{\pi}}d, \tag{6}$$

$$A_\mu = A_{\mu\perp} + \partial_\mu d, \qquad \partial_\mu A_{\mu\perp} = 0.$$

Lagrangian (6) is invariant under the transformations

$$\begin{aligned} &A_\mu \to A_\mu + \partial_\mu\alpha \; , \\ &\sigma \to \sigma - \frac{e_A}{\sqrt{\pi}}\alpha \; . \end{aligned} \tag{7}$$

The introduction of the counterterm $\Delta\mathscr{L}$ thus restores the gauge invariance and gives us a completely meaningful model. The field $\sigma(x)$ and thus the currents $j_\mu(x)$ and $j^5_\mu(x)$ become dependent on the gauge if $e_A \neq 0$.

As a four-dimensional example we consider axial electrodynamics. The Lagrangian of the model is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi + e\bar{\psi}\gamma_\mu\gamma_5\psi A^\mu \; . \tag{8}$$

Lagrangian (8) is invariant at the classical level under the transformations

$$\begin{aligned} &A_\mu \to A_\mu + \partial_\mu\alpha, \\ &\psi \to \exp(i\alpha\gamma_5 e)\psi. \end{aligned} \tag{9}$$

Model (8) has a γ_5 anomaly, whose presence causes a loss of gauge invariance.[1] An anomaly arises in this model at the single-loop level in the three-point fermion function of the axial current. Specifically, a calculation shows that there is no transversality for the three-point Green's function:

$$\begin{aligned} &(p+q)_\alpha G_3^{\alpha\mu\nu}(p,q) = -\frac{i}{6\pi^2}\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta , \\ &G_3^{\alpha\mu\nu}(p,q) = \int e^{-ipx-iqy}\langle 0|T(J_5^\mu(x)J_5^\nu(y)J_5^\alpha(0))|0\rangle d^4x d^4y , \end{aligned} \tag{10}$$

$$J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi \cdot$$

The effective action $S_{\text{eff}}(A)$ in (3) is not gauge-invariant in axial electrodynamics (8). The method proposed here for eliminating the γ_5 anomalies reduces to introducing a counterterm

$$\Delta\mathcal{L} = \frac{1}{3!}\,\frac{e^3}{2\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\partial_\mu A_\alpha \partial_\nu A_\beta \frac{1}{\Box}\,\partial^\lambda A_\lambda\,. \tag{11}$$

It is not difficult to show that the effective Lagrangian $\mathscr{L}'_{\text{eff}} = \mathscr{L}_{\text{eff}} + \Delta\mathscr{L}$ is gauge-invariant. The counterterm $\Delta\mathscr{L}$ arises only at the single-loop level. The violation of the Ward identities results from the absence of a gauge-invariant regularization in theories with chiral fermions. After an integration over the Fermi fields, on the other hand, we can work with an effective boson Lagrangian, for which gauge-invariant regularizations exist. A modified theory with a nonlocal counterterm $\Delta\mathscr{L}$ is renormalized, as follows from the fact that the contribution of the counterterm $\Delta\mathscr{L}$ to the Green's functions vanishes in the transverse gauge. Since the counterterm $\Delta\mathscr{L}$ is Hermitian, we can assume that the S matrix in this theory is unitary. Because of the nonlocal structure of counterterm (11), however, the property of locality of the theory is by no means obvious.

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