Spin waves in adsorbed H↑ and other quasi-two- and quasi-one-dimensional quantum gases

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Weakly damped spin waves can propagate in quasi-two-dimensional adsorbed H↑. The wave spectrum and the spin correlation function are calculated. Similar effects in a ^{3}He↑–^{4}He solution and in a system of electrons localized above the surface of liquid helium are discussed.

The propagation of weakly damped spin waves in gaseous H↑ and ^{3}He↑ was predicted three years ago in Ref. 1, and a theory for the effect was derived. In 1982, Lhuillier and Lalöe[2] constructed macroscopic equations for the spin dynamics of a polarized quantum gas, and these equations also imply the existence of spin oscillations. The NMR experiments carried out by Lee, Freed, *et al.*[3] were the first to reveal collective spin modes in H↑; simultaneously, a Paris group[4] experimentally discovered nuclear spin waves in ^{3}He↑. Levy and Ruckenstein[5] used a quasiparticle approach to derive a quantitative interpretation of the experimental data of Ref. 3. A detailed theory of collective quantum phenomena in Maxwellian spin-polarized gases was derived in Ref. 6. The equations of motion of the magnetization for certain other entities (binary gases, semimagnetic semiconductors, etc.) were derived and solved in Ref. 7, and a general phenomenological formulation of spin dynamics was worked out in Ref. 8.

Another interesting object for a search for collective quantum effects is quasi-two-dimensional H↑, which arises during the adsorption of atoms on a helium-covered surface. This possibility was pointed out to the author by I. P. Krylov. As Kagan *et al.* have shown,[9] adsorbed H↑ does not condense even as $T \to 0$, so that there is a temperature interval in which a Boltzmann gas begins to exhibit essentially quantum properties, i.e., under the condition $l \gg \Lambda \gg r_0$, where l is the average distance between particles, $\Lambda = \hbar/(mT)^{1/2}$ is the thermal de Broglie wavelength, r_0 is the interaction radius, on the order of atomic dimensions, and m is the mass of the particle. We consider even lower temperatures:

$$\epsilon_d^{(2)} \lesssim T \ll \hbar^2/md^2 \ll \hbar^2/mr_0^2\,, \quad d \equiv \hbar/(2m\epsilon_0)^{1/2}\,, \quad d \gg r_0\,, \tag{1}$$

where $\epsilon_d^{(2)} \sim \hbar^2 N_s/m$ is the degeneracy temperature in the two-dimensional case, N_s is the surface density of particles, d is the scale localization length of an adsorbed atom along the direction perpendicular to the surface (along the z axis), and ϵ_0 is the adsorption energy. For motion along the z axis, all the atoms are in the ground state, so that we are dealing with a typical quasi-two-dimensional situation.

A kinetic equation describing the change in the off-diagonal components of the polarized density matrix n_s, i.e., the dynamics of the transverse magnetization, is[6,7]

$$\frac{\partial n_s}{\partial t} + \frac{\mathbf{p}}{m}\nabla n_s + \frac{i}{\hbar}[\epsilon_s, n_s] = \mathrm{St}\, n_s ; \quad \epsilon_s = \frac{p^2}{2m} - \gamma \mathbf{S}\mathbf{H} + \frac{\delta E_{\rm int}}{\delta n_s}, \tag{2}$$

where **S** is the spin operator of the particle, **H** is the external magnetic field, γ is the gyromagnetic ratio, and $E_{\rm int}$ is the contribution of the interaction to the total energy of the gas. Since $r_0 \ll d \ll \Lambda$, we can calculate $E_{\rm int}$ and ϵ_s by the Fermi renormalization method, i.e., go through all the calculations using pseudopotential $\widetilde{U}$, which allows us to use perturbation theory, and then express the final result in terms of the actual s-scattering length a, which is related to $\widetilde{U}$ by the Born formula. Evaluating a diagonal matrix element of $\widetilde{U}$ with the help of the wave functions corresponding to the free motion of the atoms along the surface and to a localized state along the z axis, we find, for $r_0/\Lambda \ll 1$,

$$\langle 0 \,|\, \widetilde{U} \,|\, 0 \rangle = 4\pi a\hbar^2/mL , \qquad L^{-1} = \int_0^\infty |\psi_0(z)|^4 dz \sim d^{-1} , \tag{3}$$

where $\psi_0(z)$ is the wave function of the adsorbed particle. We consider nuclear spin waves in H$\uparrow$ caused by a difference between state populations, $|a\rangle = |\uparrow\Downarrow\rangle - \eta|\downarrow\Uparrow\rangle$ and $|b\rangle = |\uparrow\Uparrow\rangle$, where $\uparrow$ and $\Uparrow$ are the projections of the electron and proton spins, and $\eta \approx 3\times 10^{-3}$ is the small parameter of the hyperfine interaction. In first approximation, ignoring η, we can assume that the hydrogen atoms behave statistically as bosons with the proton spin of 1/2. Calculating the energy $E_{\rm int}$ with the help of (3), we find from (2)

$$\epsilon_s = A - \mathbf{S}[\gamma \mathbf{H} + (8\pi a\hbar^2/mL)\mathrm{Sp}_s \sum_p (\mathbf{S} n_s)] , \qquad a = 0{,}72\ \text{Å}, \tag{4}$$

where A is the spin-independent part of ϵ_s. For the collision integral, we restrict the analysis to gas-kinetic estimates in the τ approximation. Taking renormalization (3) into account, we can write these estimates as follows for the quasi-two-dimensional case:

$$\mathrm{St}\, n_s = -\delta n_s/\tau_2 , \qquad \tau_2^{-1} \sim (\hbar N_s/m)(a/L)^2 . \tag{5}$$

We write the density matrix n_s as a linear combination of the spin matrices:

$$n_s = a + (n_a - n_b)\mathbf{S}\vec{\mathscr{M}} + 2\mathbf{S}\vec{\lambda} , \qquad \vec{\lambda} \propto \exp(i\mathbf{k}\mathbf{r} - i\omega t). \tag{6}$$

Here n_a and n_b are the occupation numbers of the states $|a\rangle$ and $|b\rangle$, and $\vec{\mathscr{M}}$ is a unit vector along the direction of **H**. From (2), (4), and (5) we immediately find a condition under which the collisional absorption of oscillations is small:

$$1 \gtrsim |\alpha| \gg |a|/L , \qquad \alpha = (\sum_p n_a - \sum_p n_b)/N_s = (N_a - N_b)/(N_a + N_b). \tag{7}$$

Substituting (4) and (6) into (2), and working by analogy with Refs. 1, 6, and 7, we find the spectrum of weakly damped spin waves for $kv_T \ll |\Omega_{\rm int}^{(2)}|$. These waves represent a slightly nonuniform precession of the macroscopic magnetization **M** around the direction of $\vec{\mathscr{M}}$:

$$\omega = \omega' + i\omega'' = \frac{\gamma H}{\hbar} + \frac{(kv_T)^2}{\Omega^{(2)}_{\text{int}}} \left[\frac{E_a - E_b}{N_s T\alpha} - \frac{i}{\Omega^{(2)}_{\text{int}} \tau_2} \right] ,$$

$$\Omega^{(2)}_{\text{int}} = - \frac{4\pi a \hbar N_s \alpha}{m L} , \quad v_T^2 = \frac{T}{m} , \tag{8}$$

where $E_{a,b}$ is the total energy of each component in the approximation of an ideal gas,

$$E_{a,b} = \int \frac{p^2}{2m} n_{a,b} \frac{d^2 p}{(2\pi\hbar)^2} . \tag{9}$$

At $T \gg \epsilon_d^{(2)}$ we have $E_{a,b} = N_{a,b} T$, and (8) takes the form of the corresponding expression in the three-dimensional case,[1,6] with a renormalized value of $\Omega^{(2)}_{\text{int}}$.

The shape of the absorption line for the energy of the alternating magnetic field under the conditions of an NMR experiment is determined by the imaginary part of the generalized susceptibility, which is an ordinary Lorentzian curve[6]

$$\operatorname{Im} \chi(\omega, \mathbf{k}) = \frac{\gamma^2}{2\hbar} N_s |\alpha| \frac{\omega''}{(\omega - \omega')^2 + \omega''^2}, \tag{10}$$

We thus find the following expressions for the maximum intensity I and the width $\Delta\omega$ of the absorption line:

$$\Delta\omega = \omega'' \sim k^2 T / \hbar N_s \alpha^2 , \qquad I = \gamma^2 N_s |\alpha| / 2\hbar\omega'' \sim (\gamma N_s)^2 |\alpha|^3 / k^2 T . \tag{11}$$

Expressions (11) show how the parameters of the resonant-absorption line depend on N_s, T, α, and the line index k. By analogy with Ref. 6, we find from (10) the correlation function $S^{(2)}_{ik}(r) = \langle \delta M_i(\mathbf{r}_1) \delta M_k(\mathbf{r}_2) \rangle$, where $r = |\mathbf{r}_1 - \mathbf{r}_2|$, $\delta \mathbf{M} \perp \vec{\mathscr{M}}$ for $r \gg r_{\text{int}} \equiv v_T / |\Omega^{(2)}_{\text{int}}|$:

$$S^{(2)}_{ik}(r) = \delta_{ik} (\gamma N_s \alpha)^2 \frac{|a|}{L}$$

$$\times \begin{cases} K_0(r/r_{H_2}) \approx (\pi r_{H_2}/r)^{1/2} \exp(-r/r_{H_2}), & a < 0 , \\ (\pi/2) Y_0(r/r_{H_2}) \approx (\pi r_{H_2}/r)^{1/2} \sin[(r/r_{H_2}) - (\pi/4)] , & a > 0 \end{cases} \tag{12}$$

where $r_{H_2}^2 = \hbar v_T^2 / \gamma H |\Omega^{(2)}_{\text{int}}|$. If the surface is covered with a ^{3}He–^{4}He film, then we have $\epsilon_0 = 0.35$ K (Refs. 10 and 11), $d = L \approx 8$ Å, and a limiting value[9] $N_s \approx 7 \times 10^{13}$ cm^{-2}. Such high values of N_s raise the hope that it would be possible to experimentally detect the effects without having to produce a greatly extended surface. The interval of wave vectors $kv_T \ll |\Omega^{(2)}_{\text{int}}|$, in which the collisionless absorption is low and weakly damped spin waves can propagate, is significantly wider than in the three-dimensional case. At $N_s = 8 \times 10^{10}$ cm^{-2}, for example, Landau damping is slight at a spin wavelength $\lambda_s \gg 10^{-3}$ cm, so that in a resonator with a scale dimension $R \sim 1$ cm there would clearly be a large number of standing spin waves (the corresponding condition in the three-dimensional case requires a high volume density $N \sim 10^{18}$ cm^{-3}). At $T < 0.35$ K, the condition that collisional absorption be slight, $|\alpha| \gg 9 \times 10^{-2}$, clearly holds in experiments with gaseous H↑. Analogous expressions apply to other low-density quasi-two-dimensional systems, such as impurity states on a surface[12] and under bounded geometric conditions in ^{3}He–^{4}He solutions. In a quasi-one-dimensional gas of ^{3}He impurity quasiparticles (capillaries, porous materials, or localized state at vortex filaments[13]) in the case of a cylindrical geometry we have the following expression for the spin-wave spectrum:

$$\omega = \frac{\gamma H}{\hbar} + \frac{(kv_T)^2}{\Omega^{(1)}_{\text{int}}}\left[2\frac{E_+ - E_-}{N_L T\alpha} - i\frac{1}{\Omega^{(1)}_{\text{int}}\tau_1}\right], \quad \Omega^{(1)}_{\text{int}} = -\frac{32a\hbar N_L\alpha}{md^2}B; \quad kv_T \ll |\Omega^{(1)}_{\text{int}}|, \tag{13}$$

where N_L is the number of particles per unit length, $\tau_1^{-1} \sim N_L a^2\hbar^3/(md)^2 v_T$, and the coefficient B is expressed in terms of the Bessel function $J_n(x)$ by

$$B = [J_1^4(\xi_1)\xi_1^2]^{-1}\int_0^{\xi_1} J_0^4(x)\,x\,dx, \quad J_0(\xi_1) = 0. \tag{14}$$

The condition of slight damping reduces to the inequality

$$1 \gtrsim \alpha \gg (\Lambda/d)(|a|/d), \qquad \alpha = \tanh(\gamma H/2T), \tag{15}$$

and the decay of the spin correlations at $r \gg v_T/|\Omega^{(1)}_{\text{int}}|$ is described by

$$S^{(1)}_{ik}(r) = \delta_{ik}\hbar\left(\frac{2\gamma N_L\alpha}{d}\right)^2 |a| r_{H_1}\begin{cases}\exp(-r/r_{H_1}), & a<0\\ \sin(r/r_{H_1}), & a>0\end{cases}; \quad r^2_{H_1} = \frac{\hbar v_T^2}{\gamma H|\Omega^{(1)}_{\text{int}}|}, \tag{16}$$

where d is the diameter of the capillary or pore.

Spin waves can also propagate through a gas of electrons localized above the surface of liquid helium.[14] For electrons in a magnetic field, we must also incorporate the Lorentz force in (2), but for waves with $\mathbf{k} \parallel \mathscr{M}$ this force will have no effect of any sort on the wave spectrum. The dispersion relation is then determined by Eqs. (8) of Ref. 1 (to which a gap $\gamma H/\hbar$ is added), where the function ζ_p is calculated from the Coulomb interaction potential averaged over the motion along the z axis. In a real experimental situation, the spin waves would apparently be strongly damped.

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