Pion excitations in a nucleonic medium may be pertinent to the luminosity of neutron stars

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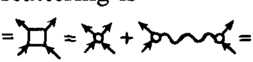
(Submitted 20 September 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **40**, No. 9, 395–398 (10 November 1984)

The luminosity per unit volume from the modified URCA process and from some new single-nucleon processes proposed here can be increased by several orders of magnitude by systematically taking into account collective effects in a dense nucleonic medium. These new single-nucleon processes involve thermal excitations of the pion field in the nucleonic medium. The increase in luminosity is probably sufficient to explain the existing experimental upper limits on the surface temperatures of neutron stars.

Recent observations of supernova remnants at the Einstein Observatory have revived interest in the problem of the cooling of neutron stars. These observations furnish upper limits on the surface temperatures, and in some cases they yield the surface temperatures of the neutron stars which are assumed to exist in the remnants of supernovae. Calculations on various reactions which lead to the cooling of neutron stars, comparisons with experiment, and bibliographies can be found in the review by Tsuruta[1] and in some other papers.[2,3] It has been shown that the processes which have customarily been considered, the most important of which is the modified URCA process $n + n \rightarrow n + p + e + \bar{\nu}$, fail to explain the experimental data, by two or three orders of magnitude in the luminosity.[4] Only a pion condensate over a broad region in a neutron star could resolve the discrepancy with experiment.[5]

In this letter we examine the role of collective effects of the nucleonic medium in the luminosity of neutron stars.

We begin with two-nucleon processes. The exact equation for the amplitude for NN scattering is[6]

$\Gamma =$  $= \Gamma_1 + \mathcal{T}_1 D_\pi \mathcal{T}_1$, (1) where Γ_1 is the NN scattering amplitude, which has no pionic excitation in the reaction channel under consideration; $\mathscr{T}_1$ is the vertex of the πN interaction, which does not contain a pion pole (Γ_1 and $\mathscr{T}_1$ incorporate nucleon-nucleon correlations); and

$$D_\pi^{-1} = \omega^2 - m_\pi^2 - \mathbf{k}^2 - \Pi(\omega, k), \qquad \hbar = c = 1 \tag{2}$$

is the pion propagator in the nucleonic medium, where $\Pi(\omega,k)$ is the temperature-dependent pion polarization operator.[6,7]

In an isotopically noninvariant medium, the nucleon correlations generally appear in different ways ($\mathscr{T}_1^{\pi^\pm} \neq \mathscr{T}_1^{\pi^0}$) for π^+, π^-, and π^0 mesons. Account should also be taken of the effect of nucleon correlations on the weak-interaction vertices. To streamline the notation, we assume identical correlations for all processes, and we use the same factor γ. We ignore the local block Γ_1 in comparison with the contribution of single-pion exchange softened by the medium. Incorporating this circumstance changes the result by $\lesssim 10\%$. Friman and Maxwell[4] approximated the nucleon-nucleon interaction as a single-pion exchange with a vacuum pion propagator $D_{\rm vac}^{-1} = \omega^2 - m_\pi^2 - \mathbf{k}^2$. The short-range repulsion was taken into account by introducing a cutoff of the single-pion exchange potential. According to Ref. 4, this evaluation of the nucleon correlations at a neutron-matter density $\rho \sim \rho_0$ (ρ_0 is the density of nuclei) changes the result by a factor of only 0.6–0.7. The primary distinction from Ref. 4 is that we are using an average pion propagator instead of a free one.

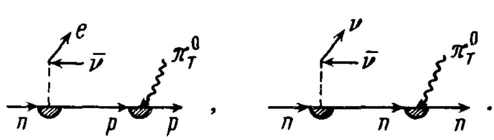
Otherwise, the calculations for the reactions $n + n \to n + p + e + \bar{\nu}$ and $n + n(p) \to n + n(p) + \bar{\nu} + \nu$ are carried out in complete analogy with the calculations of Ref. 4. As a result, we find the following expression for the luminosity per unit volume of neutron matter[1)]:

$$\epsilon_{\rm URCA} \sim 1{,}5 \times 10^{23} (\rho/\rho_0)^2 \left(\frac{m_n^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) T_9^8 \, \frac{\gamma^6 m_\pi^4}{\omega_0^4 (p_F^n)} \left[\mathrm{ergs/(cm^3 \cdot s)}\right]. \tag{3}$$

Here m_n^* and m_p^* are the effective masses of the neutron and the proton, p_F^n is the momentum of a Fermi neutron, $T_9 = (T/10^9)$ K, and $\omega_0^2(k) = -D_\pi^{-1}(\omega = 0, k)$. In (3) we have assumed ω_0^2 to be identical for $\pi^\pm$ and π^0 mesons. Substituting in the value $\omega_0^2 \sim 0.6$, we find $\epsilon_{\rm URCA} \sim 10^2\, \epsilon_{\rm URCA}$ (Ref. 4) at $\rho \sim \rho_0$ or $\epsilon_{\rm URCA} \sim 10^3 \epsilon_{\rm URCA}$ (Ref. 4) at $\rho \sim 5\rho_0$.

We have also calculated the luminosities of neutral-current processes. The contributions of these processes also increase by two or three orders of magnitude, but they remain much smaller than the contribution of the modified URCA process.

Single-nucleon processes are also possible:



(4)

The hatched vertices mean that nucleon correlations are taken into account. There are corresponding processes for charged pions. The phase volume of the single-nucleon processes is greater than that of two-nucleon processes, but processes (4) would still be suppressed by an exponential factor $N \sim \exp(-m_\pi/T)$ $(T \lesssim 0.1 m_\pi)$ if we use the vacuum pion propagator, so we will not be concerned with such processes. In a nucleonic medium, the nontrivial imaginary part of the pion polarization operator at small frequency transfer causes the exponential small factor in the occupation numbers to be replaced by a power-law small factor.[7]

We can find the luminosities of processes (4) with π^0 mesons. The square matrix element[6] summed over the final states of the pion field is

$$|M|^2 \sim \langle i|\varphi^+ \varphi|i\rangle = -2 \mathrm{Im} D_\pi^R \frac{1}{e^{\omega/T}-1}, \tag{5}$$

where D_π^R is the retarded Green's function of the pion. Using the standard rules of the diagram technique for $T \neq 0$, we find

$$\mathrm{Im}(D_{\pi^0}^R)^{-1} = \frac{f^2 k m_n^{*2}}{2\pi} \gamma^2 T \ln \frac{e^\kappa + 1}{e^\kappa + e^{-\omega/T}},$$
$$\kappa = \left(\omega + \frac{k^2}{2m_n^*}\right)^2 \frac{m_n^*}{2k^2 T} - \frac{\epsilon_F^n}{T}. \tag{6}$$

Using (5) and (6), we find the following expressions for the luminosity of processes (4), after some calculation:

$$\epsilon_{e\bar{\nu}}^{\pi^0 T} \sim 7.7\times 10^{22} \left(\frac{\rho}{\rho_0}\right)^2 \left(\frac{m_n^*}{m_n}\right)^3 \left(\frac{m_p^*}{m_p}\right) \frac{\gamma^6 m_\pi^4}{\omega_0^4 (p_F^n)} T_9^8 \sim \frac{1}{4} \epsilon_{\mathrm{URCA}}, \tag{7}$$

$$\epsilon_{\nu_e \bar{\nu}_e}^{\pi^0 T} \sim 3\times 10^{22} \left(\frac{m_n^*}{m_n}\right)^4 \left(\frac{\rho}{\rho_0}\right)^{5/3} \frac{\gamma^6 m_\pi^4}{\omega_0^4 (k_0)} T_9^8 J, \qquad J = \frac{5\omega_0^4(k_0)}{(2p_F^n)^5} \int_0^{2p_F} \frac{k^4 dk}{\omega_0^4(k)} \lesssim 1; \tag{8}$$

k_0 corresponds to the minimum of $\omega_0^2(k)$.

It can be seen from these expressions that the luminosities of single-nucleon processes at the π^0 meson and of the URCA process (3) are comparable in magnitude. Although the contribution of the neutral-current process (8) to the luminosity is slightly smaller than (7), it still increases substantially as we approach the critical pion condensation point $[\omega_0^2(k_0) = 0]$. The luminosity $\epsilon^{\pi^0} T_{\nu\nu}$ increases by a factor of three when we take into account the three neutrino species ν_e, ν_μ, and ν_τ.

We have also calculated the luminosities of the corresponding single-nucleon processes with charged pions. It turns out that the process $n + \pi^- \to n + e + \bar{\nu}$ can be important only in the early stage of the cooling of a star, at $T \sim 10^2\ T_9$, and the process $n + \pi^+ \to p + \nu + \bar{\nu}$ makes a smaller contribution.

At neutron-matter densities $\rho > \rho_c^+, \rho_c^\pm, \rho_c^0 \sim \rho_0 - 4\rho_0$, there may exist π_s^+, $\pi^\pm$, and π^0 condensates.[6] Calculations on the associated processes can be carried out by analogy with the calculations for (4), except that in this case we have $\langle i|\varphi^+ \varphi|i\rangle = a^2 (2\pi)^4 \delta(\omega - \omega_c)\delta(k - k_0)$, where a is the amplitude, and ω_c is the frequency of the pion condensate. As a result, we find

$$\epsilon_{\rm cond} = 8.8\times 10^{26}\left(\frac{m_N^*}{m_N}\right)\left(\frac{m_n^*}{m_n}\right)\frac{k_0 a^2}{m_\pi^3}\ T_9^6\,\gamma^4\,[\text{ergs/(cm}^3\cdot\text{s)}]\ . \tag{9}$$

Here the amplitude is $a = (a_{\pi_S^+}, a_{\pi^0}, a_{\pi^\pm})$, and $m_N^* = (m_n^*, m_p^*, m_n^*)$ for π_S^+, π^0, and $\pi^\pm$ condensates, respectively. In the latter case, our result agrees with a result derived previously,[5] although there is a difference in the original models. Assuming $\rho\sim 3\rho_0$, $a^2\sim 0.1m_\pi^2$, and $k_0\sim 3m_\pm$ for estimates, we find from (3), (6), and (8) that during the early stage of the cooling of a neutron star, $10^2T_9 \gtrsim T \gtrsim T_9$, processes (3) and (7) could be governing even in comparison with the processes involving a pion condensate.

We thus see that the effects associated with the propagation of a pion in a nucleonic medium can ultimately increase the luminosity of a neutron star by several orders of magnitude, even if we assume $\rho\sim\rho_0<\rho^{+,\pm,0}$, $\omega_0^2\sim m_\pi^2$. This increase is sufficient in principle to explain the existing experimental data from the Einstein Observatory, in terms of a rapid cooling of the neutron stars that arise in the remnants of supernovae. With increasing density (ρ) of the nucleonic matter, the increased softening of the pion mode (the decrease in ω_0^2) will cause our results on the luminosity per unit volume to become even higher. For this reason, even if the experimental upper limits on the surface temperatures of certain neutron stars are lowered in the future, this lowering could be attributed to a more pronounced softening of the pion mode or, ultimately, pion condensation in the interior of the these neutron stars. In order to solve the overall problem of the cooling time of neutron stars, however, it will also be necessary to take into account: (1) the decrease in the mean free path of neutrinos due to the increase in the matrix elements of the reactions studied here and (2) the increase in the heat capacity due to the additional contribution of "soft" pions. These effects, along with a possible superfluidity of nucleons, would slow the cooling of neutron stars somewhat and will be discussed in a more detailed paper.

[1)]All the luminosities written here incorporate the inverse processes.

[1]S. Tsuruta, Phys. Rep. **56**, 238 (1979).
[2]N. Nomoto and S. Tsuruta, Astrophys. J. **250**, L19 (1981).
[3]R. C. Duncan, S. D. Shapiro, and I. Wasserman, Astrophys. J. **275**, 858 (1983).
[4]B. L. Friman and O. V. Maxwell, Astrophys. J. **232**, 541 (1979).
[5]O. V. Maxwell, G. E. Brown, D. K. Campbell, *et al.*, Astrophys. J. **216**, 77 (1977).
[6]A. B. Migdal, Teoriya konechnykh fermi sistem i svoĭstva atomnykh yader (Theory of Finite Fermi Systems and Nuclear Properties), Nauka, Moscow, 1983.
[7]D. N. Voskresenskiĭ and I. N. Mishustin, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 486 (1978) [JETP Lett. **28**, 449 (1978)]; **34**, 317 (1981) [JETP Lett. **34**, 303 (1981)]; Yad. Fiz. **35**, 1139 (1982) [Sov. J. Nucl. Phys. **35**, 667 (1982)].

Translated by Dave Parsons
Edited by S. J. Amoretty