Structure of nonrotational states of deformed nuclei

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According to experimental data, the wave functions of several $K^\pi = 0_3^+, 0_4^+, 2_2^+, 2_3^+, 4_1^+$, and 4_2^+ states in deformed nuclei have large single-phonon or two-quasiparticle components, in qualitative agreement with the quasiparticle-phonon model of the nucleus and in contradiction of the model of interacting bosons.

The low-lying nonrotational states of even-even deformed nuclei are interpreted as two-quasiparticle or collective vibrational states.[1–4] In a microscopic description in the random-phase approximation, one calculates the states $\lambda\mu i$ with fixed multipolarities $\lambda\mu$, $\mu \equiv K$; the number of roots i of the secular equation is equal to the number of two-quasiparticle neutron and proton states. The first excited states ($i = 1$), $K^\pi = 2_1^+$ and 0_2^+, are collective γ and β vibrational states; then comes weakly collectivized states ($i = 2, 3, 4, \ldots$); and finally collective states that form giant resonances. The wave function is a superposition of two-quasiparticle components of a particle-hole type. For the first collective states, the normalization of the wave function is affected significantly by a large number of two-quasiparticle components, but only a small part of the space of two-quasiparticle states is taken into account.

The nonrotational states of deformed nuclei are described more accurately in the quasiparticle-phonon model of the nucleus.[5,6] In this model the wave function is written as the sum of one- and two-phonon components, and the Pauli principle is satisfied exactly in the two-phonon components. According to Ref. 6, the first excited state with a fixed value of K^π is a collective state (the contribution of the one-phonon component with $i = 1$ exceeds 80–90%). The total contribution of the one-phonon components with $i = 2, 3, \ldots$ to the wave functions of the second, third, and other states exceeds 80%. It is furthermore concluded in Ref. 6 that there are no collective two-phonon states in deformed nuclei; this conclusion does not contradict any experimental result. According to some new experimental data,[7] the state $I^\pi = 4^+$, 2.03 MeV, in ^{168}Er, which had previously been assigned $K = 4$ and which had been interpreted in Refs. 8 and 9 as a two-phonon state, has the value $K = 0$ and is not a two-phonon state.

Many calculations in recent years have been based on the model of interacting bosons.[10] In this model the description of the $K^\pi = 0^+$ and 2^+ states in deformed nuclei uses[9,11,12] an approximate classification of these states in terms of the numbers n_β and n_γ of β-and γ-vibration phonons. The first $K^\pi = 2_1^+$ γ-vibration states have $n_\gamma = 1$; the second states, 2_2^+, have $n_\gamma = 1$, $n_\beta = 1$; etc. The first excited β-vibration states, $K^\pi = O_2^+$, are characterized by $n_\beta = 1$; then we have 0_3^+ with $n_\gamma = 2$, 0_4^+ with $n_\beta = 2$, etc. In the interacting-boson model, the 2_1^+ and 0_2^+ states have roughly the same two-quasiparticle components as the one-phonon γ-and β-vibration states in

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the random-phase approximation. In a description of the γ- and β-vibration states in the microscopic approach, there is no substantial difference between the interacting-boson model and the quasiparticle-phonon model. The limitation on the phenomenological interacting-boson model is its inability to use experimental data from (dp) and (dt) reactions on the quasiparticle structure of phonons.

It can be asserted that there is a fundamental distinction between the description of the structure of several states of even-even deformed nuclei in the quasiparticle-phonon models and the interacting-boson model. In the latter, the wave functions of the $K^{\pi} = 2_2^+, 2_3^+, \ldots, 0_3^+, 0_4^+, \ldots, 4_2^+, \ldots$ states consist of components of the types $n_\gamma = 2$, $n_\beta = 2$, $n_\gamma = 1$, $n_\beta = 1$, etc., and do not contain two-quasiparticle or one-phonon components. In the quasiparticle-phonon model, the wave functions of these states contain large one-phonon $i = 2, 3, \ldots$ components and there are no large two-phonon components. From the standpoint of the microscopic approach, the interacting-boson model incorporates only that small fraction of the two-quasiparticle states which goes into the γ- and β-vibration photons. According to the quasiparticle-phonon model, the structure of these states is determined by another set of two-quasiparticle states, which are not present in the interacting-boson model. Just which of these two descriptions is more accurate will be decided by experimental data on the structure of these states, found for the most part from one- and two-nucleon transfer reactions. Experimental data[7,13-16] on ^{168}Er and ^{158,156}Gd are listed in Table I, along with results calculated for ^{168}Er and ^{158}Gd in the quasiparticle-phonon model.[6,17] In addition to the energies, we show the data illustrating the contribution of one-phonon components and their structure. For ^{168}Er and ^{158,156}Gd, the energies and structure of the γ- and β-vibration states are described quite well by the quasiparticle-phonon model and are not listed in Table I.

The nucleus for which we have the most comprehensive experimental data[13] is ^{168}Er. The rotational bands constructed from the states $K^{\pi} = 0_3^+, 0_4^+$, and 2_2^+ are highly excited in the (tp) reaction; in the $(t\alpha)$ reaction, we see evidence of a two-quasiparticle configuration $pp411\downarrow - 411\downarrow$ in the state 0_4^+, 1.834 MeV. This result indicates that the wave functions of these states have large one-phonon components. The state $K^{\pi} = 4_1^+$, 2.056 MeV, decays into a band with $K^{\pi} = 4^-$, 1.094 MeV and, according to Ref. 7, it has no large two-phonon (221, 221) components. According to calculations in the quasiparticle-phonon model,[6,17] the wave functions of all the ^{168}Er states listed in Table I have large one-phonon components. According to the calculations of Refs. 11 and 12 in the interacting-boson model, all the states listed in Table I are formed from two or three bosons and have no one-phonon components. Three bands constructed from $K^{\pi} = 0^+$ excited states have been detected experimentally[14-16] in each of the nuclei ^{158,156}Gd. In each nucleus, the 2^+0_2 and 2^+0_3 states have large values of $B(E2)$ for transitions to the ground state. According to the calculations of Refs. 15 and 16 in the interacting-boson model, either the 2^+0_2 state or the 2^+0_3 state—but not both simultaneously—has a large value of $B(E2)$. According to the experimental data, the 0_3^+ and 0_4^+ states have large one-phonon components, while the two-quasiparticle configuration $pp411\uparrow - 411\uparrow$ in the 0_4^+ state is seen clearly in ^{158}Gd in the $(t\alpha)$ reaction. The states $K^{\pi} = 4_1^+$ and 4_2^+ in each nucleus have large two-quasiparticle components, $pp411\uparrow + 413\downarrow$ and $nn521\uparrow + 523\downarrow$, respectively.

TABLE I. Structure of the states of deformed nuclei.

Nucleus	K^π	Experimental data[7,13–16] $\mathscr{E}$, MeV	Experimental data Structure	Calculations in the quasiparticle-phonon model[6,17] $\mathscr{E}$, MeV	Calculations Structure
^{168}Er	0^+_3	1.422	(tp) 10% of gr. state	1.6	202 93%;
					202: nn 521↓ − 521↓ 6% [1)]
	0^+_4	1.834	(tp) 2.4% of gr. state	1.9	203 96%;
			$(t\alpha)$ pp411↓ − 411↓ Large		203: pp411↓ − 411↓ 30%
	2^+_2	1.848	(tp) 60% of 2^+ gr. state	1.7	222 98%
					222: nn 512↑ − 521↓ 90%
	2^+_3	1.930		1.9	223 96%
	4^+_1	2.056	No large two-phonon components	1.8	441 83% {221, 221} 1%
^{158}Gd	0^+_3	1.452	$B(E2) = 19\ e^2b^2$	1.8	202 96%
			(pt) 23% of gr. state		202: nn 521↑ − 521↑ 69%
	0^+_4	1.743	$(t\alpha)$ pp411↑ − 411↑ Large	2.0	203 37%; 205 51%
					203: pp411 − 411 45%
	4^+_1	1.380	$(t\alpha)$ pp411↑ + 413↓ Large	1.5	441 96%
					441: pp411↑ + 413↓ 95%
	4^+_2	1.920	(dp) nn 521↑ + 523↓ Large	1.7	442 92%
					442: nn 521↑ + 523↓ 90%
^{156}Gd	0^+_3	1.168	$B(E2) = 15\ e^2b^2$		
			exc. in (dp) and (dt) reactions		
	0^+_4	1.715	exc. in (dt) reaction		
	2^+_2	1.828			
	4^+_1	1.510	pp411↑ + 413↓ Large		
	4^+_2	1.861	(dt) nn 521↑ + 523↓ Large		

[1)] A phonon is denoted by $\lambda\mu i$. Its contribution to the normalization of the wave function is given as a percentage. The neutron components (nn) and the proton components (pp) are specified by their asymptotic quantum numbers $Nn_z\Lambda$ (↑ for $K = \Lambda + 1/2$, ↓ for $K = \Lambda - 1/2$).

The experimental data on the $K^{\pi} = 0_3^+, 0_4^+, 2_2^+, 2_3^+, 4_1^+$, and 4_2^+ states listed in Table I demonstrate the presence in the wave functions of large one-phonon or two-quasiparticle components, which are described qualitatively correctly by the quasiparticle-phonon model but which are not present in the interacting-boson model. States of this type are observed in other nuclei, e.g., isotopes of Yb and Hf. This is also true of the $K^{\pi} = 3^+$ and other states. The agreement between the calculations carried out in the interacting-boson model and the experimental data on the energies and the values of $B(E2)$ is not satisfactory; it is necessary to find a correct description of the structure of the states. States with large one-phonon components (other than 0_2^+, 2_1^+, and 4_1^+ with a g boson) should be eliminated from calculations in the interacting-boson model. The basic shortcoming of this model is that it takes into account only a small part of the space of two-quasiparticle states, and this shortcoming cannot be overcome by an optimum choice of parameters. In spherical nuclei, states of the same type as those discussed here lie above two-phonon states, and no contradictions of the type discussed above have yet been found.

A further study of the structure of even-even deformed nuclei will require measurements of the contribution of two-quasiparticle components to the wave functions of rotational bands constructed from $K^{\pi} = 0_3, \ldots, 2_2^+, \ldots, 3_1^+, \ldots, 4_1^+, \ldots$ and other states at energies in the interval 1.5–2.5 MeV. It will also be necessary to search for two-phonon collective states.

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