X-ray surface waves in a superlattice

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It is shown that surface electromagnetic waves (SEW) with wavelengths shorter than the superlattice spacing can exist in superlattices.

1. The study of SEW on the surfaces of complex structures has recently produced considerable interest. Andreev *et al.*[1] have shown, for example, that x-ray SEW can exist in perfect crystals and can give information on the properties of the crystal surface. Surface electromagnetic waves in superlattices (SL), i.e., in artificial periodic structures consisting of alternating layers of two substances tens to hundreds of angstroms thick (see Fig. 1), is studied in Ref. 2. The wave studied in Ref. 2 is the analog of the ordinary SEW in the sense that it requires that the conditions for the existence of SEW be satisfied on the interface between any two layers of the SL[3]:

$$\epsilon_1 \epsilon_2 < 0; \quad \epsilon_1 + \epsilon_2 < 0 \tag{1}$$

just as if this interface would separate homogeneous semi-infinite media with dielectric constants ϵ_1 and ϵ_2. We shall show that aside from this, the surface of a SL can support SEW of a fundamentally different type, unrelated to conditions (1), which usually restrict the existence region of SEW to the IR and near-IR regions.

2. An electromagnetic wave which propagates along the surface of the SL is a manifestation of two effects: total internal reflection of waves incident on the boundary from the interior of the SL and Bragg reflection of waves incident on the boundary from the vacuum. To describe both of these effects simultaneously, we shall represent the SEW field in the form

$$E_y(x, z, t) = W(x) \exp(i\kappa z - i\omega t),$$

where the wave number κ is to be determined, and $W(x)$ satisfies the wave equation with the dielectric constant $\epsilon(x)$ equal to unity for $x < x_0$ and to $\epsilon_0 0(x)$ for $x > x_0$; without loss of generality, $\epsilon_0(x)$ can be assumed to be an even piecewise-continuous function (Fig. 1). We can write the solution which decays into the vacuum far from the boundary as follows:

$$W(x) \sim \exp(\sqrt{\kappa^2 - k^2}\, x), \quad x < x_0, \quad k = \omega / c = 2\pi/\lambda . \tag{2}$$

To find the solution that decays into the SL, we must represent in an approximate way the field $W(x)$ for $x > x_0$ as a superposition of two waves that interact resonantly with the SL[4]:

$$W(x) \sim (\cos qnx + \alpha_n \sin qnx)\, e^{-S_n x}, \quad x > x_0, \quad n = 1, 2, \ldots$$

$$S_n = \frac{k^2}{2nq}\sqrt{\frac{B_n^2}{4} - b_n^2}; \quad b_n = \frac{n^2 q^2 + \kappa^2}{k^2} - \mu \tag{3}$$

$$\alpha_n = \text{sign}(B_n)\sqrt{\frac{B_n - 2b_n}{B_n + 2b_n}}; \qquad q = \pi/l,$$

where l is the spacing of the SL, and μ and B_n are the coefficients in the Fourier expansion of $\epsilon_0(x)$

$$\epsilon_0(x) = \mu + \sum_{n=1}^{\infty} B_n \cos 2qnx.$$

For the case of $\epsilon_0(x)$ which is illustrated in Fig. 1, we have

$$\mu = \epsilon_2 + \beta(\epsilon_1 - \epsilon_2); \quad B_n = 2(\epsilon_1 - \epsilon_2)\frac{\sin \pi n\beta}{\pi n}, \quad n = 1, 2, \ldots .$$

Expression (3) is valid for $|B_n| \ll 2q^2n^2k^{-2}$.

Solutions (2) and (3) describe a field which decays on both sides of the boundary of the SL, if

$$\kappa > k \text{ and } \mu - \frac{n^2q^2}{k^2} - \frac{|B_n|}{2} < \frac{\kappa^2}{k^2} < \mu - \frac{n^2q^2}{k^2} + \frac{|B_n|}{2}. \tag{4}$$

We introduce below the auxiliary parameter $\phi_n = \arctan \alpha_n$. We can then write the dispersion equation obtained by joining solutions (2) and (3) at the boundary of the SL and the additional conditions (4) as follows:

$$\cos^2(\phi_n - \pi n\beta_0)\left[1 + \frac{B_n}{2(\mu-1)}\cos 2\phi_n\right] = \frac{\pi^2n^2}{k^2l^2(\mu-1)}; \tan(\phi_n - \pi n\beta_0) > 0, \tag{5}$$

where $\beta_0 = x_0/l$ (see Fig. 1).

The wave number κ can be expressed with the help of the solution of Eq. (5) in terms of the frequency ω of the SEW and the physical parameters of the SL ϵ_1, ϵ_2, l, β, and β_0 as follows:

$$\left(\frac{\kappa}{k}\right)^2 = \mu - \left(\frac{\pi n}{kl}\right)^2 + \frac{B_n}{2}\cos 2\phi_n. \tag{6}$$

It can be shown that dispersion equation (5) has for each value of n no more than one solution; the conditions for the existence of this solution with $B_n < 0$ can be written in the form of inequalities

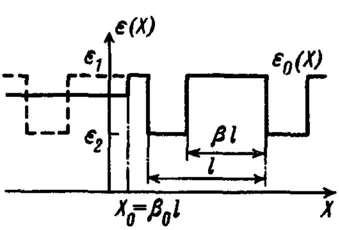

FIG. 1. Dielectric constant $\epsilon(x)$ (solid curve) and the basic parameters of the SL.

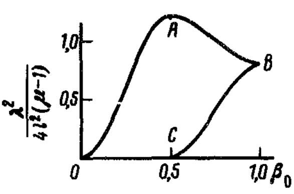

FIG. 2. The curve $OABCO$ bounds the existence region of the first surface mode ($n = 1$) for the case $B_1/[2(\mu-1)] = -0.2$.

$$\frac{n^2\lambda^2}{4l^2(\mu-1)} < \left[1 - \frac{B_n}{2(\mu-1)}\right]\sin^2\pi n\beta_0 \quad \text{for } \tan\pi n\beta_0 > 0,$$

$$\left[1 + \frac{B_n}{2(\mu-1)}\right]\cos^2\pi n\beta_0 < \frac{n^2\lambda^2}{4l^2(\mu-1)} < 1 + \frac{B_n}{2(\mu-1)}\cos 2\pi n\beta_0 \quad \text{for } \tan\pi n\beta_0 < 0. \tag{7}$$

In (7) we have $\lambda = 2\pi/k = 2\pi c/\omega$. The existence region is shown in Fig. 2.

All of the relations obtained above are independent of the specific form of the periodic distribution of the dielectric constant.

3. We shall now consider the basic properties of SEW at the boundary of the SL, which follow from expressions (5)–(7).

1. There is a minimum spacing of the SL

$$l_{\min} \simeq \frac{\lambda}{2}(\mu-1)^{-1/2} \tag{8}$$

such that for $l < l_{\min}$ there can be no surface electromagnetic waves.

2. For $l > l_{\min}$ the existence of SEW is determined by the value of β_0 or, in other words, by the thickness of the uppermost layer of the SL.

3. In the case of a weakly absorbing SL ($\mathrm{IM}\mu \ll |\mathrm{Re}\, B_n|$), the damping of the SEW along the boundary of the SL can be represented as

$$\mathrm{Im}\,\kappa \simeq \frac{k}{2}\,\mathrm{Im}\left(\mu + \frac{B_n}{2}\cos 2\phi_n\right)\tan(\phi_n - \pi n\beta_0)\left[\tan(\phi_n - \pi n\beta_0) + \frac{2l^2}{n^2\lambda^2}\mathrm{Re}\, B_n \sin 2\phi_n \times \cos^2(\phi_n - \pi n\beta_0)\right]^{-1}. \tag{9}$$

If the parameters of the SL lie near the boundary AB of the existence region of SEW (Fig. 2), then $\phi_n \simeq \pi n\beta_0$ and the damping of the SEW along the boundary may be small.

4. We see from (4) and (5) that for the existence of SEW the value of the dielectric constant must be $\mu > 1$. This condition ($\mu > 1$), which is much less stringent than condition (1) for the ordinary SEW, can be satisfied over a broad range of wavelengths, down to the soft x-ray region.

5. The electromagnetic field of the SEW (3) encompasses many layers of the SL and does not have an analog among the usual SEW. The latter circumstance is also

evident from the fact that the SEW studied by us in the SL is an E wave, whereas the usual SEW can be only H-type waves if there is a single interface between homogeneous media.[3] It is evident that H-type x-ray SEW, which generally exist on the surface of the SL, vanish as $|\epsilon_1 - \epsilon_2| \to 0$, since the average dielectric constant μ is > -1 in the x-ray region.

Let us consider an example. In the case of an SL consisting of layers of carbon and aluminum with the parameters $l = 440$ Å, $\beta = 0.15$, $\beta_0 = 0.75$, in the wavelength band $(\lambda - \lambda_{cr})/\lambda_{cr} \sim 10^{-3}$–$10^{-4}$ [where $\lambda_{cr} \approx 170$ Å is determined from Eq. (8)], according to Eq. (9), we have the following expression for the mean free path of SEW: $L = (\mathrm{Im}\kappa)^{-1} > 1 - 10$ mm.

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