Light scattering by spin waves in superfluid ^{3}He*A*

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A new mechanism for light scattering in the *A* phase of superfluid ^{3}He, involving a scattering by fluctuations of the anisotropy of the magnetic susceptibility, is analyzed. The extinction coefficient is calculated for this mechanism for scattering by spin waves.

1. The scattering of light by spin waves is of major interest for research on magnetic materials. Study of this scattering can lead to a better understanding of magnetooptical phenomena, presents additional possibilities for testing theories, and requires an extremely subtle experimental technique (see the review by Borovik-Romanov and Kreines,[1] for example).

There are several mechanisms for the scattering of light by spin waves. In most cases the scattering would be one by fluctuations of the dielectric tensor. At the microscopic level, this mechanism involves electric dipole transitions and thus corresponds to a cross section $\propto (\omega/c)^4$, where ω is the frequency of light, and c is the velocity of light. Borovik-Romanov and Kreines[1] set forth a phenomenological theory for this scattering and discussed the extensive experimental consequences. Another interesting mechanism for scattering by spin waves was proposed by Bass and Kaganov.[2] In this mechanism it is assumed that the magnetic moment is saturated, so that the entire effect of the incident light reduces to a rotation of this moment. For this scattering mechanism the cross section would have the behavior $\propto [(\gamma\omega)^2/c^4]$ (where γ is the gyromagnetic ratio), so that the corresponding contribution would usually be much smaller than that caused by fluctuations of the dielectric constant.

The superfluid phase of He^3, which have come under intense study in recent years, exhibit some unique magnetic properties. We are thus led to the question of scattering of light in these phases. In this connection it would be important to study fluctuations of the spin and of the order parameter in superfluid ^3He. This problem was studied in Ref. 3 by the technique of Mori projection operators. In the present letter, in contrast with Ref. 3, we use Landau and Lifshitz's method of hydrodynamic fluctuations,[4] so that by using the Leggett-Takagi spin-dynamics equations[5] we can derive explicit expressions for the correlation functions and make some concrete numerical estimates. It is important to note that the relationship between the fluctuations of the order parameter and the dielectric tensor in superfluid He^3 is not known, and in any case the corresponding proportionality coefficients are anomalously small.[6] In the isotropic *B* phase of He^3, we are thus left with essentially only the scattering involving the rotation of the spin vector.[2] Significantly, in the anisotropic *A* phase of He^3 there can be a new mechanism for the scattering of light waves with a wavelength less than or on the order of the coherence length, $\sim 10^{-5}$ cm, by fluctuations of the magnetic-susceptibility tensor. These fluctuations are directly related to the order parameter of the *A* phase:

$$\chi_{ij} = \chi_0 \delta_{ij} + \chi_1 d_i d_j , \quad (1)$$

where **d** is the unit spin-anisotropy vector of the *A* phase. At the microscopic level, this mechanism is related to a magnetic dipole radiation, so that the corresponding scattering cross section has the behavior $\propto (\omega/c)^4$, as in the electric-dipole case. Making use of the symmetry of Maxwell's equation with respect to electrical and magnetic phenomena, we can immediately write an expression for the derivative of the extinction coefficient with respect to the frequency:

$$\frac{dh}{d\omega} = \frac{\omega^4}{16\pi^2 c^4} < \delta\chi_{ij}\, \delta\chi_{mn} > p_i p_m p'_j p'_n , \quad (2)$$

where $\delta\chi_{ij}$ is a fluctuation of the tensor χ_{ij} associated with a fluctuation of the order parameter, and **p** and **p**′ are the polarization vectors (polarization of the magnetic field!) in the incident and scattered waves.

2. The spin dynamics in superfluid ^3He is described by the Leggett-Takagi equations,[5] which take the following form for ^{3}He*A*:

$$\frac{\partial \mathbf{s}}{\partial t} = g_D (\mathbf{dl})(\mathbf{d}\times\mathbf{l}) + K(\mathbf{d}\times\Delta\mathbf{d}) + D\Delta\mathbf{s};$$

$$\frac{\partial \mathbf{d}}{\partial t} = \gamma^2 \chi_0^{-1}(\mathbf{s}\times\mathbf{d}) - \Gamma_{\parallel}(\mathbf{dl})[\mathbf{d}\times[\mathbf{d}\times\mathbf{l}]], \quad (3)$$

where *s* is the spin, **l** is the unit orbital-anisotropy vector, g_D is the constant of the dipole-dipole interaction, $\Gamma_{\parallel}$ is the width of the longitudinal NMR line, *K* is an elastic constant, and *D* is the spin-diffusion coefficient introduced in Ref. 7. For definiteness, we restrict the discussion to a configuration in which transverse spin waves can propagate. Specifically, we assume that the equilibrium spin vector is directed along the *z* axis of the laboratory coordinate system, that the vector **l** is directed along the *y* axis, and that the vector **d** lies in the *xy* plane, making an angle α with the *y* axis. The fluctuation then reduces to a change in *s* and in the angle α. We furthermore assume that all variables depend on only the coordinate z and the time. System of equations (3) can then be rewritten

$$\frac{\partial s}{\partial t} = - g_D \,\delta\alpha + K \frac{\partial^2}{\partial z^2}\,\delta\alpha + D \frac{\partial^2}{\partial z^2} s\,,$$

$$\frac{\partial \alpha}{\partial t} = \gamma^2 \chi_0^{-1} s - \Gamma_{\parallel}\,\delta\alpha\,. \tag{4}$$

To calculate the correlation functions in (1), we need to introduce random forces in (4). Analysis of the dissipative terms in the Leggett-Takagi equations shows that the random forces must be introduced in the following way. In the first equation, the random source f is related to spin diffusion, while in the second (F) it is related to Leggett-Takagi dissipation. To determine the correlation functions $\langle ff\rangle$, $\langle Ff\rangle$, and $\langle FF\rangle$, we must write the entropy production,[5] in accordance with the general procedure[4]:

$$\frac{\partial \sigma}{\partial t} = \frac{1}{T}\gamma^{-2}\chi_0\Gamma_{\parallel}\Omega^{-2}(\delta\alpha)^2 + \frac{1}{T}\gamma^2\chi_0^{-1}D\left(\frac{\partial s}{\partial z}\right)^2 \tag{5}$$

(Ω is the Leggett frequency).

From (4) and (5) we have

$$\langle ff\rangle = 2\gamma^{-2}\chi_0 DT;\quad \langle FF\rangle = \frac{\gamma^2\chi_0^{-1}}{\Omega^2}\Gamma_{\parallel}T;\quad \langle Ff\rangle = 0\,. \tag{6}$$

It is now a simple matter to find the correlation function that we need from (4) and (6):

$$\langle\delta\alpha^2\rangle_{q,\omega} = \frac{\left[D^2q^4\frac{\Gamma_{\parallel}}{\Omega^2} + \frac{\Gamma_{\parallel}}{\Omega^2}\omega^2 + Dq^2\right]\gamma^2\chi_0^{-1}T}{\{\omega^2 - [\Omega^2 + q^2(\gamma^2\chi_0^{-1}K + \Gamma_{\parallel}D)]\}^2 + \omega^2(\Gamma_{\parallel} + Dq^2)^2}\,. \tag{7}$$

The poles of the denominator in (7) determine the spin-wave spectrum in the A phase of He^3. A spin diffusion must be introduced in order to correctly determine the Onsager coefficients and thus the correlation functions of the random forces.

From (1) and (7) we can easily estimate the total extinction coefficient associated with the scattering by spin waves due to fluctuations of the magnetic susceptibility:

$$h \sim \left(\frac{\omega}{c}\right)^4 \chi_1^2 \frac{\gamma^2\chi_0^{-1}T}{\Gamma_{\parallel}\Omega^2} \sim 10^{-12}\ \mathrm{cm}^{-1}$$

(at a pressure of 30 bar, $1 - T/T_c \sim 0.4$ for spin waves with a length on the order of the dipole length). This value is indisputably small, but with the rapid progress in experimental techniques there is the hope that such values of h could be measured in He^3A in the near future. We might note in this connection that the Mandel'shtam-Brillouin scattering by second sound in He^4 predicted long ago by Ginzburg,[8] appeared at one time to be virtually impossible to study experimentally, but today waves of this type can be observed well on the basis of light scattering.[9] In principle, it would also be experimentally feasible[10] to observe the light scattering by zeroth sound in the Fermi liquid He^3 ($h \sim 10^{-9}$ cm^{-1}) predicted by Abrikosov and Khalatnikov.[10]

We note in conclusion that fluctuations may be significantly larger near nonequilibrium quasistationary states. One such state is the state of the B phase with the maximum dipole energy (the corresponding angle of the order parameter is π).

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