Numerical simulation of two-dimensional Langmuir turbulence

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A two-dimensional Langmuir turbulence has been simulated numerically. The results show that the energy of the Langmuir oscillations in this type of turbulence is localized in collapsing cavities which are constantly produced and which are "burned up" by Landau damping. The integral characteristics of the quasisteady state found from the simulation agree satisfactorily with an analytic model.

The analytic model of strong plasma turbulence proposed in Refs. 1 and 2 is based on Langmuir collapse[3] as the primary mechanism for the short-wave conversion of the plasma waves. Numerical simulations of turbulence have previously been restricted to the one-dimensional case[4] or the situation near the threshold.[5] We know quite well that the Langmuir collapse of an isolated cavity can occur in a two- or three-dimensional geometry, so a quantitative comparison with the analytic model of turbulence requires a numerical simulation in at least two dimensions. In this letter we report a numerical simulation of a two-dimensional (x, y) turbulence produced by a pump wave with electric vector oriented along the X axis and with a one-dimensional amplitude

$$E_{0x} = \frac{1}{2} E_0 e^{-i\omega_{p_0} t} + \text{c. c.}, \qquad E_0 = \text{const} .$$

This pump simulates either an electromagnetic wave in the plasma-resonance region or long plasma waves excited by a beam in a plasma. The turbulence is simulated on the basis of the Zakharov equations, modified to allow for the pump wave and for the wave absorption by resonant particles in the short-wave part of the spectrum for both of the modes that form the turbulence: the high-frequency plasma mode and the slow mode of quasineutral density perturbations. This system of equations has been used on several occasions previously (see Ref. 6, for example). In contrast with Ref. 6, we used as initial conditions random distributions of the high-frequency electric field and the density with mean square amplitudes

$$\frac{< E^2 >(t=0)}{8\pi n_0 T} \simeq 10^{-4} \quad \text{and} \quad \frac{\sqrt{< \delta n^2 >(t=0)}}{n_0} \simeq 10^{-6},$$

These distributions give rise to the turbulence. The energy density of the pump wave was varied over the range $E_0^2/8\pi n_0 T = 1.5\times 10^{-3}$–$1.3\times 10^{-2}$. As a result of the modulational instability of the pump wave, long plasma waves are excited with wave numbers satisfying the condition $k \lesssim k_0$, where

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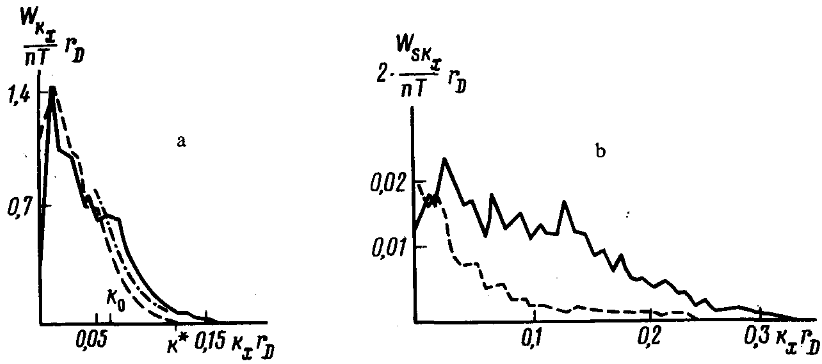

FIG. 1. a—Spectrum (k_x dependence) of the energy of the Langmuir waves averaged over the y direction, $w_{k_x} = (1/8\pi \int_0^{L_y}(dy/L_y)|\int_0^{L_x} E(x,y)(e^{ik_x x}/\sqrt{L_x})dx|^2$, for a nonisothermal plasma (solid curve) and an isothermal plasma (dashed curve) [the dot-dashed curve shows the case $W_{k_x} = \int_0^\infty W_k(dk_y/k) \sim (1/k_x^2)$; b—$k_x$ dependence of the energy of the sound waves averaged over the y direction, $W_{sk_x} = (n_0T/2)\int_0^{L_y}(dy/L_y)|\int_0^{L_x}(\delta n/n_0)(e^{ik_x x}/\sqrt{L_x})dx|^2$, for a nonisothermal plasma (solid curve) and an isothermal plasma (dashed curve).

$$k_0 = \frac{1}{3r_D}\sqrt{\frac{W_L}{n_0T}}$$

($W_L = \Sigma_{\mathbf{k}}|E_{\mathbf{k}}|^2/8\pi$ is the energy of the plasma waves). The subsequent pumping of the waves toward shorter wavelengths is related to the collapse phenomenon. Figure 1 shows the spectra of the high- and low-frequency waves found in the numerical simulation. In the plasma-wave spectrum, we can clearly distinguish three regions: a long-wave region of the modulational instability (the source region), an inertial interval, and the absorption region. In the source region, the waves that are not trapped in the cavities are dominant. According to Ref. 1, for such waves we have the pressure balance (cf. Fig. 2)

$$\frac{\sqrt{<\delta n^2>}}{n_0} = \frac{\sqrt{\sum_{k<k_0}|\delta n_{\mathbf{k}}|^2}}{n_0} \simeq \frac{W_L}{n_0T} \tag{1}$$

(the summation is over wave numbers in the source region). Waves trapped in cavities (packets of high-frequency energy with the plasma displaced from them) are entrained in the collapse process. Upon collapse, the energy influx from the pump is "disconnected" from the cavity,[6] and we find the following spectrum for the plasma waves in the inertial interval for the two-dimensional case from the expression for the self-similar collapse of an isolated cavity[1]:

$$|E_k|^2\,dk \sim \frac{dk}{k^2}, \qquad k = \sqrt{k_x^2 + k_y^2}\,. \tag{2}$$

This expression agrees satisfactorily with the numerical simulation. The spectrum of plasma waves in the long-wave region is anisotropic, stretched out along the direction of the pump. For various values of the pump amplitude, the ratio $\langle k_{xL}\rangle/\langle k_{yL}\rangle$ varies

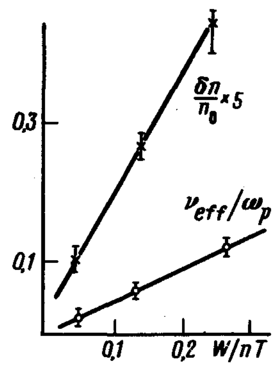

FIG. 2. The effective collision rate $\nu_{\rm eff}$ and the long-wave density perturbations, $\sqrt{\langle \delta n^2 \rangle}/n_0$, versus the energy in a steady-state turbulence for an isothermal plasma.

over the interval 1.5–1.3, where the vector $\langle \mathbf{k}_L \rangle$ is determined from the relation $\langle \mathbf{k}_L \rangle = \Sigma_{k<k_0} \mathbf{k}|E_k|^2/\Sigma_{k<k_0}|E_k|^2$. On the other hand, the spectrum found for the inertial interval and the absorption region in the numerical simulation is essentially isotropic, and the ratio $\langle k_{xs} \rangle / \langle k_{ys} \rangle$ is equal to unity within 5–10% ($\langle \mathbf{k}_s \rangle$ is defined as the average wave vector of the waves, but in the short-wave part of the spectrum, $k > k_0$). The isotropy of the short-wave part of the spectrum is maintained by the random orientation of the cavities in the turbulence, although each cavity is of a dipole nature and therefore anisotropic (Fig. 3).

A quasisteady state of the turbulence is reached as the result of a balance between the energy influx into the turbulence from the pump wave and the absorption of the energy of the short plasma waves by resonant electrons. Correspondingly, the effective collision rate $\nu_{\rm eff}$—a measure of the rate of energy influx into the turbulence—is determined from the expression

$$\nu_{\rm eff} = 2 \sum_k \Gamma_k \, |E_k|^2 / E_0^2 , \tag{3}$$

where Γ_k is the rate of the Landau damping of the plasma waves, calculated under the assumption of a Maxwellian distribution of resonant electrons. According to the analytic model of Refs. 1 and 2, as the pump amplitude is varied, the quasisteady level of $\nu_{\rm eff}$ varies linearly with W (Fig. 2). The quasisteady turbulence assumes a dynamic nature because of the continuous production, collapse, and subsequent "burn-up" of cavities with the plasmons trapped in them. Figure 3 shows a typical distribution of the field and the density in the quasisteady state. The disruption of the balance between the plasma pressure and the rf pressure causes a cavity with the burned-up plasmons to become the source of rather short sound waves. The dynamics of the short-wave conversion of the plasma waves depends to a large extent on the nature of the damping of the acoustic mode. In an isothermal plasma ($T_e = T_i$), the sound is damped strongly as a result of interaction with resonant ions; the damping rate is $\gamma(k) \simeq \omega_p \sqrt{(m/M)} k r_D$. In this case the quasisteady level of short-wave sound,

$$W_s \simeq \frac{n_0 T}{2} \sum_{k>k_0} \frac{|\delta n_{\mathbf{k}}|^2}{n_0^2}$$

is determined from the balance equation proposed in Ref. 1:

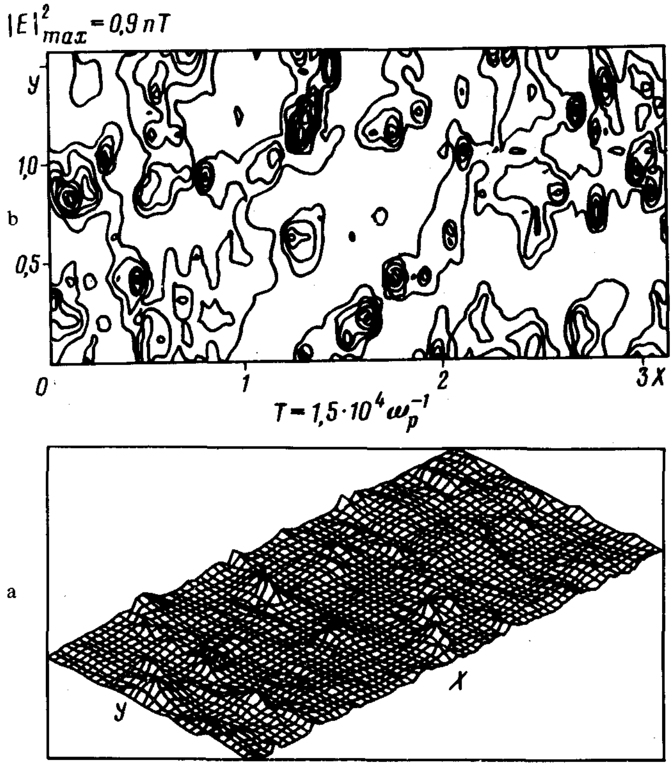

FIG. 3. Spatial distribution of the rf energy $|E(x, y)|^2$ in a steady-state turbulence in a nonisothermal plasma for a pump level $E_0^2/8\pi n_0 T \approx 5\times 10^{-3}$. a—Isothermal projection; b—contour map.

$$2\gamma(k = \langle k_s \rangle) W_s \simeq \nu_{\text{eff}} \frac{E_0^2}{8\pi} \beta \langle k_s \rangle^2 r_D^2 \ . \tag{4}$$

The quantity $\beta \langle k_s \rangle^2 r_D^2$ is the fraction of the energy of the plasma waves which is converted into sound in the final stage of the cavity collapse. The coefficient $\beta \simeq 0.2$ is found from the numerical simulation of the collapse of a two-dimensional cavity. In the case $T_e = T_i$, the sound level found from the numerical simulation agrees satisfactorily with that found from balance equation (4). The basic mechanism for the short-wave conversion of the plasma waves, which creates an energy flux $\nu_{\text{eff}} E_0^2/8\pi$ over the spectrum, in this case is actually the collapse of the cavities with the plasmons, accompanied by the continuous excitation of the acoustic mode. The Langmuir energy flux in the collapsing cavities can be written $I = B(W_L)\gamma_{\text{mod}} W_L$, where $\gamma_{\text{mod}} = \omega_p \sqrt{(m/3M)(W_L/n_0 T)}$ is the growth rate of the modulational instability, which determines the collapse time of an isolated cavity, $\tau_{\text{coll}} = 1/\gamma_{\text{mod}}$. The factor $B(W_L)$ is a measure of the efficiency at which the plasma waves are trapped in the course of the collapse. This factor, which is not present in the original version of the analytic model,[1,2] may be related to the finite time required for a cavity to reach the self-similar collapse regime. In the numerical simulation, we have $B \sim W_L^{1/2}$, so that the balance equation $I = \nu_{\text{eff}}(E_0^2/8\pi)$ leads to a functional dependence $W \sim E_0^2$, which is also found in the numerical results.

In a nonisothermal plasma ($T_e \gg T_i$), the sound waves are damped solely by the interaction with resonant electrons, and the damping rate is substantially lower, $\gamma^e(k) = \omega_p(m/M)kr_D$. Correspondingly, the energy in the short sound waves is higher, but it remains roughly ten times lower than that found from balance equation (4) under these conditions. This result is attributable to the fact that during the buildup of sound waves the collapse of cavities with plasmons—a process that continuously creates sound waves—ceases to be the primary source of the short-wave conversion of Langmuir energy. In this case, there are two possible mechanisms associated with the excitation of sound for the short-wave pumping of the plasmons. One of these mechanisms, proposed in Ref. 7, is the conversion of plasmons from the source region directly into a short-wave mode, which is immediately absorbed by the particles. This mechanism is important if the spectrum of sound waves has a maximum in the short-wave region, as it does in the case of a one-dimensional turbulence.[4] For the spectrum in Fig. 1, the growth rate for this process is $\gamma_{\text{conv}} \ll \gamma_{\text{mod}}$, and the other mechanism is important: the multistep pumping of plasmons over various scale dimensions in the inertial interval as a result of the sound waves created during the collapse. The scale time for the pumping from the source region ($k \sim k_0$) into the absorption region ($k \sim k^*$) is

$$\tau_{TR} \simeq 36\pi \frac{r_D^4}{\nu_{\text{eff}}} \int_{k_0}^{k^*} dk k^2 \left[\int_0^{2k} d\kappa \frac{|\delta n_\kappa|^2/n_0^2}{\sqrt{1-\kappa^2/4k^2}} \right]^{-1} . \tag{5}$$

In a nonisothermal plasma ($T_e \gg T_i$), the level of the sound waves in the numerical simulation is determined from the condition $\tau_{TR}^{-1}(W) = \gamma_{\text{mod}}(W)$, so that this mechanism for the short-wave pumping of the plasmons is the primary mechanism, and the collapse is allowed only to the extent required to maintain the appropriate sound level.

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