Generation of isothermal density perturbations in an inflationary universe

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Isothermal density perturbations with an amplitude sufficient for the subsequent formation of galaxies may be generated along with adiabatic density perturbations in an inflationary universe.

1. The appearance of the initial density perturbations that are required for the subsequent formation of galaxies is one of the most difficult and important problems of cosmology today. Significant progress has recently been achieved in this study. It has been shown that in the stage of exponential expansion in the inflationary-universe scenario perturbations of the density ρ are generated with a spectrum $[\delta\rho(l)/\rho]$, which is essentially independent of the scale length of the perturbations, l (a so-called flat spectrum or Zel'dovich spectrum[1]). In several models, the amplitude of the perturbations can reach the value $\delta\rho/\rho \sim 10^{-4}$, which is required for the formation of galaxies.[2] In essentially all the studies of the generation of perturbations in an inflationary universe, however, the discussion has been restricted to adiabatic perturbations, with the implication that if perturbations of other types (isothermal, for example) do arise, then they will always be negligible in comparison with adiabatic perturbations.[3] This circumstance, combined with the conclusion of an almost absolutely flat perturbation spectrum and the conclusion that the density of the universe is near the critical value, has led several investigators to doubt that it would be possible to reconcile the predictions of certain versions of the inflationary-universe scenario with the theory of the formation of the large-scale structure of the universe[4] and with the small value of the quadrupole anisotropy of the background radiation.[5] In the present letter we show that isothermal perturbations of respectable amplitude can be generated in addition to adiabatic perturbations in a wide range of theories, and the spectrum of these isothermal perturbations may be slightly different from a flat spectrum.

2. Let us first review, in a slightly simplified form, the basic mechanism for the generation of inhomogeneities in an inflationary universe.[2] We consider a universe with a scaling factor $a \sim e^{Ht}$ which contains a scalar field φ with an effective potential $V(\varphi)$ and a small effective mass $m^2(\varphi) = d^2V/d\varphi^2 \ll H^2$. In this case, fluctuations in the field φ are generated, and as time elapses these fluctuations acquire a spectrum

$$< (\delta\varphi)^2 > \sim \frac{H^2}{4\pi^2} \int_k^H \left(\frac{k}{H}\right)^{2m^2/3H^2} d\ln k \qquad (1)$$

in the long-wave region,[6,7] $k \ll H$. As a result, inhomogeneities of the density ρ_φ of the field φ arise with an amplitude

$$\frac{\delta\rho_\varphi(k)}{\rho_\varphi} \sim \frac{\delta\varphi(k)}{\varphi}\,, \tag{2}$$

where φ is the value of the classical field φ at the time under consideration, and $\delta\varphi(k)$ is the amplitude of the fluctuations of the field φ in a logarithmic scale of the momentum k, given by

$$\delta\varphi(k) \sim \frac{H}{2\pi}\left(\frac{k}{H}\right)^{m^2/3H^2}. \tag{3}$$

If $m^2 \ll H^2$, we find density perturbations with a flat spectrum. The subsequent evolution of these perturbations depends on how the field φ interacts with the other types of elementary particles.

3. Many types of scalar fields figure in the modern theory of elementary particles, and some of these fields may have essentially no interactions with particles of other types (the so-called hidden or latent sector of the theory; see Ref. 8, for example). As a very simple example, we consider the theory of two fields φ_1 and φ_2, which do not interact with each other and which have effective potentials $V(\varphi_i) = (m_i^2/2)\,\varphi_i^2 + (\lambda_i/4)\,\varphi_i^4$ with $\lambda_1 \ll \lambda_2$ and $m_1 \ll m_2$. We assume that the field φ_2, in contrast with φ_1, is in the hidden sector. As was shown in Ref. 9, if $\varphi_i \gtrsim M_p$ (where $M_p \sim 10^{19}$ GeV is the Planck mass), the fields φ_i slide down very slowly to the minima of $V(\varphi_i)$ because of the appearance of terms with a "friction" $3H\dot{\varphi}_i$ in the equations for the fields φ_i, and at this time the universe is expanding exponentially. It usually turns out that the greater curvature of the potential $V(\varphi_2)$ causes this slow-slide regime to end first for the field φ_2, when the Hubble "constant" $H(\varphi_1, \varphi_2) = \sqrt{(8\pi/M_p^2)\,(V(\varphi_1) + V(\varphi_2))}$ is smaller than the effective mass $m(\varphi_2) = \sqrt{d^2V/d\varphi_2^2}$, which has the value $\sqrt{3\lambda_2}\varphi_2$ at large values of φ_2. Consequently, the lighter of the fields φ_i is responsible for the last stage of the inflation. The inflation ends at $\varphi_1 \sim (M_p/3)$, at which we have[9] $H \sim \sqrt{\lambda_1} M_p$. In this case it follows from (2) that[2]

$$\frac{\delta\rho_1}{\rho_1} \sim \sqrt{\lambda_1}. \tag{4}$$

After the decay of the particles of the field φ_1 and the heating of the universe, the corresponding inhomogeneities lead to inhomogeneities in the temperature distribution of the gas of ultra-relativistic particles, i.e., to *adiabatic* perturbations with an amplitude determined by the *smaller* of the constants λ_i in (4).

Because of the term with the friction $3H\dot{\varphi}_2$, the field φ_2 rapidly slides down from the value $\varphi_2 \sim M_p$ not to zero but to that value at which the curvature of the potential $V(\varphi_2)$ is several times smaller than $H^2(\varphi_1)$:

$$m^2(\varphi_2) \sim \lambda_2\varphi_2^2 \sim CH^2(\varphi_1) = \frac{8\pi C}{3M_p^2}\,V(\varphi_1), \tag{5}$$

where $C < 1$. Then comes another regime of a slow decrease of the field φ_2, which gives way to a rapid decrease only after the end of inflation. In this regime, fluctuations of the field φ_2 are generated in addition to fluctuations of φ_1, and the former lead [according to (2), (3), and (5)] to perturbations of the density of this field on the order of

$$\frac{\delta\rho_2}{\rho_2} \sim \frac{\delta\varphi_2}{\varphi_2} \sim \sqrt{\lambda_2}. \tag{6}$$

The density perturbations in (6) are not related to the temperature perturbations in (4) and in this sense are isothermal. In the early stage, the total density of matter is determined by the relativistic gas that arises from the decay of the field φ_2 (Refs. 7 and 9). Later, however, the fraction of the energy that is in the field φ_2 increases, since, by assumption, the particles of φ_2 do not decay into other particles, and their energy density in the inflationary universe falls off more slowly than the energy of the relativistic gas. It can be shown that, with $m_2/\sqrt{\lambda_2} \sim 10^{10}$ GeV, for example, it is the φ_2 particles that must now represent the primary component of the density of matter in the universe.[10] In this case the basic density perturbations, which are *isothermal*, are determined by the *larger* of the constants λ_i [see (4) and (6)].

An analogous mechanism leading to the formation of perturbations operates in more complicated models, such as the axion model; in contradiction of an assertion in Ref. 3, the isothermal perturbations in these models can also be large. A particularly interesting effect that arises in the axion models is the cutoff of the flat spectrum of isothermal perturbations at large wavelengths. The effective potential in models of this sort is periodic in φ: $V(\varphi) \sim 1 - \cos(\varphi/\varphi_0)$. The contribution of short-wave fluctuations with $k \gtrsim k_0$ to $\overline{\delta\varphi} = \sqrt{\langle(\delta\rho)^2\rangle}$ in (1) begins to exceed $\pi\varphi_0$—the maximum effective amplitude of the field φ in the theory—at $k_0 \sim H \exp\left(-\frac{4\pi^4\varphi_0^2}{H^2}\right)$. As a result, expression (2) for $\delta\rho(k)/\rho$ is no longer valid at $k \ll k_0$, and the spectrum $\delta\rho(k)/\rho$ is cut off at $k \ll k_0$. This effect can be understood most easily in the case of a homogeneous "perturbation" of the field φ: $k \to 0$. Clearly with a dispersion $\sqrt{\langle(\delta\varphi)^2\rangle} \gg \pi\varphi_0$, there would be essentially equal probabilities for the appearance of any value of φ between $-\pi\varphi_0$ and $\pi\varphi_0$. In this case, because of the periodicity of $V(\varphi)$, the spectrum $\delta\rho(k)/\rho$ becomes completely independent of the arbitrary constant part $\delta\varphi(0)$ of the field φ. The rate of decrease of $\delta\rho(k)/\rho$ at large scale dimensions $l \gtrsim k_0^{-1}$ is model-dependent, determined by the relationship between φ_0 and H.

The conclusion that we wish to reach is that although the basic perturbations that are generated in the inflationary-universe scenario are usually adiabatic perturbations with a flat spectrum, there are some important and interesting exceptions to this "rule." I believe that the results above will permit a more flexible approach to the problem of the formation of galaxies and to the question of the anisotropy of the background radiation.

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