Soliton regime of spin diffusion in the bistable state

G. B. Teĭtel'baum
Kazan Physicotechnical Institute, Academy of Sciences of the USSR

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It is shown that a transition from one stationary magnetization to another under conditions of bistability resulting from radio-frequency (rf) pumping occurs due to the propagation of a solitonlike magnetization front.

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In paramagnetic spin systems with an S-like dependence of the amplitude of rf magnetization on the intensity of the rf field (characteristic), two stable stationary states with different values of magnetization can occur.[1] Such behavior is characteristic, for example, for nuclear spin systems with a dynamic shift of the nuclear magnetic resonance (NMR) frequency.[2,3] For such systems, we shall show how the transition of the magnetization from one stationary state to another occurs under conditions of nonlinear spin diffusion.

We shall examine nuclear spins with a dynamic shift of the NMR frequency, arising due to their indirect interaction via the electron spins. In easy-plane antiferromagnets, such shifts can greatly exceed the width of the NMR line over a wide temperature range.[3] Let the z axis coincide with the orientation of the constant magnetic field, and let the pumping field lie in the xy plane. If it is assumed that the transverse magnetization (of a single sublattice in the case of antiferromagnets) $m_{\pm}(t) = \bar{m}_{\pm} e^{\mp i\omega t}$, then the slowly varying amplitude and longitudinal magnetization $m_{\pm}$ satisfy the modified Bloch equations

$$\dot{\bar{m}}_{\pm} = \pm i(\omega - \omega_s + \omega_p \frac{m_z}{m_0} \pm \frac{i}{T_2}) \bar{m}_{\pm} \pm i \gamma m_z h_{\pm}, \tag{1}$$

$$m_z = \frac{\gamma}{2i}(\bar{m}_- h_+ - \bar{m}_+ h_-) + \frac{m_0 - m_z}{T_1}. \tag{2}$$

Here $h_{\pm}$ is the amplitude of the field with frequency ω amplified due to the hyperfine interaction, ω_s is the spin resonance frequency, ω_p is the dynamic frequency shift parameter, m_0 is the equilibrium nuclear magnetization, and γ is the nuclear gyromagnetic ratio. If the transverse and longitudinal relaxation times are related by the conventional relation $T_2 \ll T_1$, the adiabatic approximation can be used and an equation can be obtained for the longitudinal magnetization ($x = m_z/m_0$)

$$T_1 \dot{x} = 1 - x - \alpha \nu^2 x [\nu^2 + (\Delta + x)^2]^{-1}. \tag{3}$$

This is essentially the equation of balance of the power absorbed by the spins and the power transferred to the lattice. Here $\alpha = \gamma^2 h^2 T_1 T_2$ is the population parameter, and $\nu = (T_2 \omega_p)^{-1}$ and $\Delta = (\omega_s - \omega)/\omega_p$ are the width of the line and the detuning in units

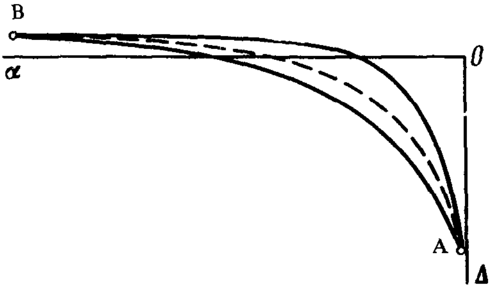

FIG. 1. Phase diagram of stationary magnetization in the pumping power-detuning plane. The solid curves bound the region of bistability. The dashed curve is the curve of phase equilibrium. A and B are the critical points.

of the shift. It follows that at $\nu \ll 1$, bistable states exist for some α and Δ. In the α, Δ plane (see Fig. 1), they correspond to points lying inside the boomerang-shaped region, which is bounded by the lines of absolute instability of stationary states x_1 and x_2 (the transverse coordinates are uniquely given by the corresponding x_i). In the vicinity of the unique critical points $A\,(8/3\sqrt{3}\nu;\ \sqrt{3}\nu - 1)$ and $B\,(27/64\nu^2, 1/8)$, these lines are given by the equations

$$-\frac{3}{4}\delta\alpha = \delta\Delta \pm 2^{1/3}\, 3^{-5/4}\, \nu^{-1/4} (\delta\Delta)^{3/2}$$

near A and

$$\nu^2 \delta\alpha = -\frac{9}{4}\delta\Delta \pm (-2\delta\Delta)^{3/2}$$

near B, where $\delta\alpha$ and $\delta\Delta$ are the deviations from the critical values α_c and Δ_c.

The transition between stationary values of magnetization occurs via the magnetization corresponding to the increasing section of the dependence $m_z(h)$, on which the uniform state is unstable. Such a transition could be caused by a large fluctuation or it can be stimulated by an external action. As a result, there appears a transitional front, whose geometry is determined by fluctuations with the largest growth increment. To describe the propagation of nonuniform magnetization, we shall include in Eqs. (1)–(3) the spin diffusion with the coefficient D, which occurs due to the secular part of the Suhl-Nakamura interaction. Analysis of these equations, together with the Maxwell equations, in the magnetostatic approximation reveals that the largest increment occurs for perturbations that are nonuniform along the constant magnetic field. The point is that only these perturbations do not cause perturbations of the transverse magnetic field which suppress the instability. This permits reducing the problem to a one-dimensional problem. We note that this picture differs from the situation in semiconductors, where the S-shaped current-voltage characteristic leads to the appearance of current filaments, rather than layers.[4]

The shock-wave-type solutions of interest to us depend only on the combination $\xi = z + vt$, where v is the velocity along the negative z axis. In accordance with (3), the stationary distributions of magnetization in a system of coordinates moving with velocity v are given by the solutions of the equation

$$DT_1 \frac{\partial^2 x}{\partial \zeta^2} - vT_1 \frac{\partial x}{\partial \zeta} - \frac{\delta F(x)}{\delta x} = 0. \tag{4}$$

An analogous equation in mechanics describes the motion of a particle with mass DT_1 and with friction $-vT_1$ in a potential field

$$U(x) = -F(x) = x - \frac{x^2}{2} - \frac{\alpha \nu^2}{2} \ln[(\Delta + x)^2 + \nu^2]$$

$$+ \alpha \Delta \nu \ \mathrm{arc\,tg}\left(\frac{\Delta + x}{\nu}\right), \tag{5}$$

where the variable ζ plays the role of time. We are interested in the solution corresponding to the motion of a particle which in the limit $\zeta \to -\infty$ is located at the point x_1 and arrives at the point x_2 in the limit $\zeta \to \infty$. In the phase plane x, $\partial x/\partial \zeta$, it corresponds to a separatrix passing from the saddle point x_1 to the saddle point x_2. This exact case is realized with a single value of the bifurcation parameter vT_1, which determines the velocity of the front. Near the critical points A and B, the potential has the simple form

$$U(x) = \mathrm{const} + b\left[\tilde{h}\delta x + a_1 \frac{(\delta x)^2}{2} - \frac{(\delta x)^4}{2}\right], \tag{6}$$

and the stationary stable values $x_{1,2} = x_c \pm \sqrt{a_1}$. Here

$$\tilde{h} = \frac{4}{9} \lambda_c^2 \delta \Delta - \nu^2 \left(1 - \frac{2}{3}\lambda_c\right) \delta\alpha; \ b = (\nu^2 + \lambda_c^2 / 9)^{-1}; \ \lambda_c = \Delta_c + 1;$$

$$a_1 = \frac{2}{3} \delta \Delta \lambda_c (3 - 4\lambda_c) / (3 - 2\lambda_c); x_c = (1 - 2\Delta_c)/3.$$

To find the velocity, we equate the change in the potential energy with the transition from x_1 to x_2 to the frictional losses

$$2\tilde{h}\sqrt{a_1} = -v\, T_1 \int_{x_1}^{x_2} \frac{dx}{d\zeta}\, dx. \tag{7}$$

In the linear (with respect to $\tilde{h}$) approximation, we substitute here the value $\partial x/\partial \zeta = \sqrt{b/2DT_1}\,[a_1 - (\delta x)^2]$ calculated for $\tilde{h} = 0$ and we obtain

$$v = -3\sqrt{Db/2T_1}\ (\tilde{h}/a_1). \tag{8}$$

It is evident that for $\tilde{h} > 0$, the magnetization x_1 propagates along the z axis and the magnetization x_2 is displaced. For $\tilde{h} < 0$, the opposite picture occurs. It is easy to show by using standard methods[5–7] that such solutions are stable in the entire bistability region. The characteristic free path of such a soliton $\sim \sqrt{v/w}$ is determined by the distance it traverses until it meets an antisoliton. Here w is the probability of the formation of a critical nucleation center per unit length per unit time.

On the line $\tilde{h}=0$ (see Fig. 1), there is an equilibrium of phases, which, in plates with thickness d oriented parallel to a constant magnetic field, can lead to the formation of a domain structure with a period $\sim\sqrt{r_0 d}$. The distribution of magnetization in the domain wall is given by

$$\delta x(z)=\sqrt{a_1}\ \mathrm{th}\left(\frac{z-z_0}{r_0}\right),$$

where $r_0=\sqrt{2DT_1/a_1 b}$ is its width. Thus, by changing the rf pumping power or the detuning of the frequency, the nuclear magnetization can be transferred from one state to another.

These states are separated in frequency by a distance greatly exceeding the width of the resonance line. For this reason, the change of the relative intensity of the corresponding NMR signals in time, characteristic for displacement of the front, can be observed by passing through the NMR with an additional weak field. Another method for observing the change is to investigate the shift of the frequency of the antiferromagnetic resonance.

The formation of such solitons in a medium that is spatially uniform in the absence of rf pumping is typical of paramagnetic systems with an S-shaped characteristic.

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