Properties of a 2D electron gas with lifted spectral degeneracy

Yu. A. Bychkov and É. I. Rashba

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

The properties of a 2D electron gas in which the twofold spin degeneracy of the spectrum has been lifted by a perturbation $\widehat{H}_{so} = \alpha[\vec{\sigma} \times \mathbf{k}]\vec{\nu}$ are analyzed. The coefficient α is found from experimental data reported recently by Stein *et al.* and Stormer *et al.* on combined and cyclotron resonances at a heterojunction. The magnetic susceptibility is also studied.

If a crystal has a single high-symmetry axis (at least threefold) and an invariant vector $\vec{\nu}$ is oriented along this axis, the Hamiltonian of the spin-orbit interaction of an electron is

$$\hat{H}_{so} = \alpha[\vec{\sigma} \times \mathbf{k}]\vec{\nu}, \tag{1}$$

where $\vec{\sigma}$ are the Pauli matrices. In crystals with the wurtzite structure, the unit vector $\vec{\nu}$ is directed along the **c** hexagonal axis,[3,4] and at heterojunctions it is directed along the normal.[5] The operator $\widehat{H}_{so}$ lifts the two fold spin degeneracy of the zone at $\mathbf{k} \neq 0$ and determines the spin-orbit splitting of the zones near $\mathbf{k} = 0$. Zone splitting has been calculated for several systems[6–8]; the methods and errors of these calculations are analyzed in Refs. 5 and 9. This splitting was recently observed at GaAs-Al$_x$Ga$_{1-x}$As heterostructures in *n*- and *p*-type layers.[1,2]

In this letter we show that the coefficient α can be extracted from the experimental data reported by Stein *et al.*[1] and Stormer *et al.*[2] by making use of the theory of Ref. 10. We will also analyze the magnetic susceptibility χ of a 2D gas, which has some interesting properties because of the presence of the H_{so} term (χ was studied in Ref. 11 for corresponding 3D systems). The possibility of measuring χ in a 2D gas was demonstrated in Refs. 12 and 13.

The Hamiltonian $\widehat{H}$ and the dispersion law $E^{\pm}(k)$ (Fig. 1a) are

$$\hat{H} = \frac{\hbar^2}{2m^*} k^2 + \alpha[\vec{\sigma} \times \mathbf{k}]\vec{\nu}, \quad E^{\pm}(k) = \frac{\hbar^2 k^2}{2m^*} \pm \alpha k. \tag{2}$$

Here **k** is the 2D quasimomentum, and $k = |\mathbf{k}|$. The value of $E^-(k)$ reaches a minimum on a circle,[3,4] which is a loop of extrema. The radius of this circle is $k_0 = \alpha m^*/\hbar^2$, and we have $E^-(k_0) \equiv -\Delta = -(m^*\alpha^2/2\hbar^2)$. The spectrum in a magnetic field $\mathbf{H} \| \vec{\nu}$ is described by[10]

$$E_s^{\pm} = \hbar\omega^* \epsilon_s, \quad \epsilon_0 = \delta, \quad \epsilon_s^{\pm} = s \pm \sqrt{\delta^2 + \gamma^2 s}, \quad s \geq 1, \tag{3}$$

where the values of s are integers,

$$\omega^* = \frac{eH}{m^*c}, \quad \gamma = 2\sqrt{\Delta/\hbar\omega^*}, \quad \delta = \frac{1}{2} - \beta, \quad \beta = \frac{m^*}{2m_s}, \quad m_s = \frac{2m_0}{g}. \quad (4)$$

Here m_0 is the mass of the free electron, and m_s and g are the "spin mass" and the g-factor. Transitions involving a change in s within a single branch of the spectrum (+ or −) correspond to a cyclotron resonance; at $\hbar\omega^* \gg \Delta$, the frequencies of this resonance approach ω^*. Transitions between branches (with or without a change in s) excited by a microwave electric field are called "combined resonances." At $\hbar\omega^* \gg \Delta$, the transition at the spin-resonance frequency, $h\nu_0(H) = 2\beta\hbar\omega^*$ is the strongest.[10]

Stein et al.[1] observed a combined resonance of 2D electrons at the boundary of a heterojunction under the condition $\gamma^2 s \ll 1$. It follows from (3) that the frequency of the spin transition for the Landau level $N = 1$ (with an energy $\approx 3\hbar\omega^*/2$) under these conditions is

$$\nu(H) \approx \nu_0(H) - \frac{6\Delta}{\delta h} \text{sign } g. \quad (5)$$

This linear dependence $\nu(H)$ agrees with the experimental data in Fig. 2 of Ref. 1. A positive ν intercept implies $g < 0$ (according to Ref. 14). For electrons in GaAs we have $|\beta| \ll 1$ and thus $\delta \approx 1/2$. According to Ref. 1, we then have $12\Delta/h = 7.8 \pm 1.5$ GHz for sample 2; we then find $\Delta \approx 2.5 \times 10^{-6}$ eV and $\alpha \approx 2.5 \times 10^{-10}$ eV·cm.

Stormer et al.[2] have observed a cyclotron resonance of 2D holes at the boundary of a heterojunction. They found two absorption peaks (Fig. 3 in Ref. 2) with a linear dependence of the frequency ω on H in the field interval 30–80 kG. From the slopes they found the masses $m_1^* = 0.38m_0$ and $m_2^* = 0.60m_0$. The size-effect splitting of the valence band causes the hole spectrum in the channel to consist of two types of sub-bands: with angular-momentum projections of 1/2 and 3/2. For GaAs with the ordinary (001) orientation of the heterojunction, the point symmetry group of the interface (subgroup T_d) is C_{2v}. The fourfold symmetry is lost as a result of the filling of the last atomic layer at the boundary of the heterojunction. If we ignore this effect, which has yet to be observed, and if we proceed phenomenologically, we need to replace C_{2v} by one of the groups C_{4v}, D_{2d}, or D_4. These groups are isomorphic, and for subbands of both types they lead to a Hamiltonian which is unitarily equivalent to H_{so} in Eq. (1). A linear H dependence of $\hbar\omega_s^{\pm} = E_{s+1}^{\pm} - E_s^{\pm}$ with highly different masses can be found only in the semiclassical limit with $\gamma^2 s \gg \delta$. If the Fermi energy satisfies $E_F \gg \Delta$, then

$$\omega_c^{\pm}(H) \approx \omega^*(H) \left\{ 1 \pm \sqrt{\frac{m^*\Delta}{\pi\hbar^2 n}} \right\}, \quad (6)$$

where n is the hole density. From the experimental values of m_1^*, m_2^*, $n \approx 5 \times 10^{11}$ cm^{-2}, and $m^* \approx 0.5m_0$ we find $\Delta \approx 10^{-4}$ eV and $\alpha \approx 0.6 \times 10^{-9}$ eV·cm. The conditions for the applicability of expression (6) are satisfied, but just barely. A completely convincing distinction of the effect of $\hat{H}_{so}$ and the effect of a nonparabolic deviation would require measurements over a broader H interval and a comparison with Eq. (3).
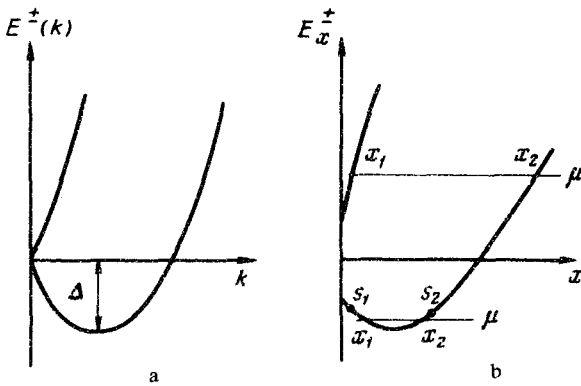
FIG. 1. a—The dispersion law $E^\pm(k)$ [Eq. (2)]; b—the $E_x^\pm$ dependence [Eq. (3)].

We thus see that the combined resonance and the cyclotron resonance yield the values of the parameters α and Δ. We turn now to a calculation of χ, which is also sensitive to these parameters and which should make it possible to determine them.

In a weak magnetic field $\gamma^2 \gg 1$ in the orientation $\mathbf{H} \| \vec{\nu}$, the thermodynamic potential Ω at the temperature $T = 0$ is

$$
\Omega = \frac{r^* H^2}{2\pi} \left\{ -\delta^2 \sqrt{\frac{\mu + \Delta}{\Delta}} \cdot \theta(-\mu) - (\delta^2 - \delta + \tfrac{1}{6})\theta(\mu) \right.
$$

$$
\left. + \frac{1}{2}B_2(x_1) \frac{\sqrt{\mu + \Delta}}{|\sqrt{\Delta} - \sqrt{\mu + \Delta}|} + \frac{1}{2}B_2(x_2) \frac{\sqrt{\mu + \Delta}}{\sqrt{\Delta} + \sqrt{\mu + \Delta}} \right\} . \tag{7}
$$

Here $\theta(\xi)$ is 1 or 0 at $\xi > 0$ and $\xi < 0$; $B_2(x)$ is the Bernoulli polynomial (with a unit period); $r^* = e^2/m^*c^2$; and μ is the chemical potential. The quantities x_1 and x_2 are the roots of the equation $E_x^\pm = \mu$ (Fig. 1b). Equation (7) holds at $x_1 \gg 1$ (i.e., if $|\mu| \gg \sqrt{\hbar\omega^*\Delta}$). If $|\mu|$ is small, then Ω will increase to $\Omega \sim r^* H^2 \gamma$. The susceptibility $\chi = -(\partial\Omega/\partial H)_\mu$ is positive at $-\Delta < \mu < 0$ because of the first term in expression (7). Superimposed on the monotonic increase are oscillations due to the last two terms in (7). If $|\mu| \lesssim \sqrt{\hbar\omega^*\Delta}$, then $\chi \sim -r^*\gamma$. At $\mu > 0$, the oscillations are superimposed on the constant component from the second term.

At constant n, the susceptibility χ undergoes jumps at times corresponding to the beginning of the filling of a new $(N + 1)$st quantum level:

$$
\chi_\Delta = n_L N (E_{N+1} - E_N) / H^2. \tag{8}
$$

The levels N are indexed in order of increasing energy, and the sequence of indices includes both spectral branches; for the lowest level, $N = 1$. For the dispersion law of Fig. 1a, there are jumps of yet another type: at times at which, as H is varied, a partially filled level $(s_2$, say) becomes coincident with a vacant level $(s_1$; Fig. 1b). In this case we have

$$\chi_\Delta = \frac{2\Delta}{H^2}[n - n_L(\dot{s}_2 - s_1 - 1)]\left\{ \frac{s_1}{\sqrt{\delta^2 + \gamma^2 s_1}} - \frac{s_2}{\sqrt{\delta^2 + \gamma^2 s_2}} \right\}. \tag{9}$$

At a constant value of μ we find instead of (8)

$$\chi_\Delta = \frac{n_L}{H} \frac{dE_N}{dH}. \tag{10}$$

Equations (8)–(10) hold for an arbitrary magnitude of H. Features analogous to (8) should naturally appear in the kinetic coefficients also.

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