Particle creation in a tunneling universe

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The problem of the behavior of quantized fields in the process of tunneling of a closed homogeneous isotropic universe is examined. It is shown that tunneling of the universe from the Friedman regime to the de Sitter regime is accompanied by catastrophic creation of particles.

In recent years, a great deal of attention has been devoted to problems related to tunneling of a closed universe as a whole.[1-3] This problem attracts special interest in connection with attempts to describe the quantum creation of the universe out of "nothing" (see Ref. 2 and references therein). The purpose of this paper is to discuss the problem of the behavior of quantized fields and the process of a tunneling transition of a closed homogeneous isotropic universe with a positive cosmological constant out of the Friedman regime (or out of "nothing") into a de Sitter-type regime. We shall use the approximation in which tunneling is described quasiclassically and we shall ignore the inverse effect of the particles on the tunneling process. We shall show that creation of particles accompanying tunneling from the Friedmann regime is catastrophic at least in the model of a massive scalar field with conformal coupling; the probability of creation increases exponentially with penetration into the classically forbidden region.

Having in mind the approximations indicated above, we shall write the Wheeler-de Witt equation[4] under the assumption that all degrees of freedom, except the scale factor R and the particle fields $\varphi(x)$, are unimportant,

$$\left(\frac{1}{24}\frac{\partial^2}{\partial R^2} - 6R^2 + \Lambda R^4 + \pi_T + H_M[\hat{\varphi}(x), \hat{\pi}_\varphi(x): R]\right)|\Phi\rangle = 0\,; \tag{1}$$

(below, we shall assume that $\Lambda \ll 1$). Here we have set $\hbar = c = M_{Pl} = 1$, so that all quantities are measured in Planckian units, we also introduced into the equation the constant π_T, which arises, for example, when the universe is filled with an ideal gas with an equation of state $p = \rho/3$ (Ref. 5) (in the classical solutions of the Einstein equations, $\pi_T = \rho R^4$). The choice of the order of cofactors in (1) is not important in our approximation (in connection with this, see Ref. 6). In Eq. (1), H_M is the Hamiltonian operator of particles in conformal time; in the case of a massive scalar field with conformal coupling, we have

$$H_M = \sum_{k=1}^{\infty}\sum_{l=0}^{k-1}\sum_{m=0}^{l} H_{klm}\,, \tag{2}$$

where H_{klm} is the Hamiltonian for the mode with quantum numbers k, l, m

$$H_{klm} = \frac{1}{2}\hat{\pi}_{klm}^2 + \frac{1}{2}(k^2 + \mu^2 R^2)\hat{\varphi}_{klm}^2 ;$$

$$[\hat{\pi}_{klm}, \hat{\varphi}_{k'l'm'}] = -i\delta_{kk'}\delta_{ll'}\delta_{mm'} .$$

The state vector of the universe $|\Phi(R)\rangle$ for each fixed R is a vector in a Hilbert space of particle states (for example, $|\Phi\rangle$ can be assumed a Fock column vector in which the elements depend on R).

Ignoring the particle energy, Eq. (1) coincides with the Schrodinger equation for a "particle" with "energy" π_T in a "potential" $V(R) = 6R^2 - \Lambda R^4$. For $1 \ll \pi_T \ll \Lambda^{-1}$, the universe can tunnel out of the classically accessible region of small R (Friedman regime) into the region of large R (de Sitter regime).[3] For $\pi_T \lesssim 1$, the region of R corresponds to a universe with a Planck radius, and tunneling occurs out of this state ("nothing," "mini-universe"[4,6] "quantum puff"[7]) into an expanding de Sitter universe.[1)]

If H_M does not depend on R, then Eq. (1) can be solved by the method of separation of variables. It is clear that for fixed π_T and high conformal energy of particles (eigenvalues of the Hamiltonian H_M), the probability of penetrating beneath the barrier is exponentially large compared with tunneling with small conformal energy.[2)] If H_M depends on R, then particle creation is possible (transitions to excited levels if H_M) during tunneling, which then leads to an exponential increase of the wave function beneath the barrier. Particle creation is thus a favorable process for tunneling. The physical reason for this fact is that the increase in the energy of particles indicates simultaneous increase in the modulus of the negative energy of the gravitational field (the total energy of the closed universe is zero), giving rise to the higher probability for subbarrier penetration.

To find the equation for the quantum theory of fields in the tunneling universe, we shall proceed in analogy with the derivation[8] of the Tomonaga-Schwinger equation for the state vector of matter in an external classical gravitational field from the Wheeler–de Witt equation. We shall write the state vector in the form $|\Phi(R) = e^{-S(R)}|\Psi(R)\rangle$, where s satisfies the equation $(dS/dR)^2 = 24(V - \pi_T)$. In the approximation described above, we obtain

$$-\frac{1}{12}\frac{dS}{dR}\frac{\partial|\Psi\rangle}{\partial R} + H_M|\Psi\rangle = 0.$$

We introduce the "Euclidean conformal time" and switch from the variable R to the variable τ, such that $12(dR/d\tau) = (dS/dR)$. We thus find

$$\frac{\partial|\Psi\rangle}{\partial\tau} = H_M|\Psi\rangle . \tag{3}$$

Equation (3) is our basic equation, which is valid if the total energy of particles in much lower than $V(R)$. The exponential increase of $|\Psi(\tau)\rangle$, which follows from (3), corresponds at nonzero conformal energy to the increase in the tunneling probability discussed above.[3)]

We shall discuss the solution of Eq. (3) in the model (2) in the case of tunneling out of the Friedman regime. Since modes with different quantum numbers behave independently of ($|\Psi\rangle = \prod_{k,l,m} |\Psi^{klm}(\tau)\rangle$), it is sufficient to solve Eq. (3) for fixed k, l, and m. We shall expand $|\Psi^{klm}(\tau)\rangle$ with respect to the characteristic states of the instantaneous Hamiltonian $H_{klm}(\tau)$: $|\Psi^{klm}\rangle = \Sigma_{n=0}^{\infty} C_n^{(k)} e^{\int \omega_k d\tau/2} |\Psi_n^{klm}\rangle$ (we drop the indices l and m in $C_n^{(k)}$). The n-particle amplitudes $C_n^{(k)}$ satisfy the equation

$$\frac{\partial C_n^{(k)}}{\partial \tau} = n\omega_k C_n^{(k)} + \sum_{n' \neq n} \frac{C_{n'}^{(k)}}{\omega_k(n-n')} < \Psi_n^{klm}(\tau) | \frac{\partial H_{klm}}{\partial \tau} | \Psi_{n'}^{klm}(\tau) >, \tag{4}$$

where $\omega_k^2 = k^2 + \mu^2 R^2$. Let, for example, there be a vacuum state immediately in front of the barrier: $C_n^{(k)}(\tau = 0) = \delta_{n0}$. Solving Eq. (4) by iterations, ignoring particle annihilation (it can be shown that this is admissible), we obtain

$$C_{2n}(\tau) = \frac{\sqrt{(2n)!}}{n!} \times \left[e^{2\int_0^\tau \omega_k d\tau'} \int_0^\tau e^{-2\int_0^{\tau_1} \omega_k d\tau''} \frac{\mu^2 R(\tau_1)}{2\omega_k^2(\tau_1)} \frac{dR(\tau_1)}{d\tau_1} d\tau_1 \right]^n, \tag{5}$$

so that all multiparticle amplitudes are of the order of unity at

$$\tau \approx \sqrt{\frac{2(R-R_1)}{R_1}} = \frac{1}{\epsilon_k R_1} \ln \frac{\epsilon_k^2 R_1}{\mu}, \quad \epsilon_k^2 = \mu^2 + k^2 R_1^{-2}, \tag{6}$$

where R_1 is the maximum radius of the classical Friedman expansion (it is assumed that $R_1 \gg \mu^{-1}$).

It is evident from (5) and (6) that particle creation proceeds all the more intensely the higher the wave number k and for large k, it becomes catastrophic at the beginning of the tunneling transition. For this reason, the inverse effect of particles on the transition process must not be ignored. Allowance for the inverse effect presumably leads to the fact that the classical expansion in the de Sitter regime begins from the point of the maximum of the "potential" $V(R)$, and in this case the conformal energy of the created particles at $R = R_{\max}$ is $V(R_{\max}) - \pi_T$. We also expect that the inverse effect will increase the amplitude of the transition: instead of being suppressed by a factor $\exp(-\text{const}/\Lambda)$,[3] we can expect that the amplitude will have a power-law dependence on Λ^{-1}. The creation of particles with nonzero k, l, and m can also lead to inhomogeneities and anisotropy of the universe at the beginning of the de Sitter stage.

Application of the method described above for solving Eq. (3) to the case of tunneling out of "nothing" leads to analogous results. However, the definition of the vacuum for small R must be made more precise in this case.

In this paper we did not consider the problem of renormalization, which presumably is required in constructing a systematic quantum theory of fields in the tunneling universe. This problem requires further study.

In conclusion, we note that the technique described above may be useful in studying particle creation in other models of the tunneling universe. The catastrophic creation of particles seems to be a rather general property of tunneling in quantum gravitation.

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[1)] The problem of creation of the universe out of "nothing" was also examined in this model by A. A. Starobinskiĭ.

[2)] It follows that if particle creation accompanied classical expansion and if the state of the Universe was, immediately prior to the barrier, a superposition of states with a different energy of matter and the corresponding amplitudes decreased according to a power law as a function of energy, then the most probable state beyond the barrier would be a classical expansion, beginning from the peak of the "potential" $V(R)$, while the amplitude for passing through the barrier would be small in a power-law sense but not exponentially small.

[3)] It is not difficult to generalize Eq. (3) to the case of a different (nonconformal) choice of time. We note that in the case of nongravitational tunneling in field theory[9] it can be shown that the analogous equation has a negative sign on the right side, and catastrophic particle creation does not occur.

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