Critical behavior of the Josephson frequency of superconducting composites

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The time-varying Josephson effect in superconducting composites near the percolation threshold is analyzed. A new effect is predicted: a dependence of the frequency of the Josephson emission on the concentration of the superconducting phase. A new exponent of percolation theory is introduced: a critical exponent of the concentration dependence of the Josephson frequency. This exponent is derived on the basis of a weak-link model.

The superconducting properties of inhomogeneous systems have attracted a great deal of interest, because the numerous high-T_c ceramics are very inhomogeneous[1] and also because of practical applications of superconducting composites.[2,3]

In this letter we point out a new aspect of the time-varying Josephson effect in superconducting composites, and we determine the critical exponents of the frequency of Josephson emission (the dependence of the frequency of this emission on the concentration of the superconducting phase).

We consider superconducting composites consisting of elements with a minimum size a_0. We assume that the concentration of the superconducting phase, p, is above the percolation threshold p_c ($p > p_c$). We assume that in the normal state the conductivity (σ_1) of phase 1 (e.g., a metal which goes superconducting as the temperature is lowered) is much higher than that (σ_2) of phase 2 (e.g., an insulator or a semiconductor): $\sigma_1 \gg \sigma_2$.

When the concentration of phase 1 is below the percolation threshold, there are regions of phase 2 between the drops of phase 1 (Ref. 4), and the Josephson effect is observable. At $p = 1$ the distance between contacts is large, and Josephson generation is not observed. It would be logical to suggest that near the percolation threshold at $p > p_c$, the Josephson frequency, like other properties, varies in a critical fashion: $\omega \sim \tau^{-\alpha}$. Below we find support for this suggestion on the basis of a weak-link model, and we derive the exponent α.

The weak-link model, used to analyze the structure of very inhomogeneous composites, was proposed in Refs. 4 and 5. According to this model, when the concentration of the conducting phase is above the percolation threshold, the very inhomogeneous medium consists of blocks of large cross section (of phase 1) which are connected to each other by long, narrow bridges (also of phase 1) and thin, curved "interlayers" (of phase 2), connected in parallel. When the current exceeds the critical value, the bridges and interlayers go into the normal state, while the blocks may remain super-

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conducting. A Josephson generation may be observed under these conditions. In this model one can determine the characteristic dimensions of the element of the inhomogeneous structure [which are in a volume element with a typical size L, where $L \sim a_0 \tau^{-\nu}$, a_0 is the minimum length scale of the problem, $\tau = (p - p_c)/p_c$, and ν is the critical exponent]. At $p > p_c$, for example, one can determine the bridge length l (the bridge is a single-strand filament of phase 1, with most of the resistance) and the interlayer area a (the interlayer of phase 2, with a characteristic thickness a_0, is connected parallel to the bridge). Both l and s depend on the concentration of phase 1. They are given by

$$l \sim a_0 \tau^{-t+\nu}, \quad s \sim a_0^2 \tau^{-q-\nu}, \tag{1}$$

where t and q are critical exponents.

The weak-link model gives a correct description of the effective conductivity σ^e of a highly inhomogeneous medium in the normal state. The expression for σ^e is the same as that proposed in Ref. 6. Within second-order terms, it is

$$\sigma^e \sim \sigma_1 \tau^t (A_0 + A_1(\sigma_2/\sigma_1)\tau^{-t/s}), \tag{2}$$

where A_0, $A_1 \sim 1$.

This model has been used to determine the concentration dependence and the field dependence of the galvanomagnetic and thermogalvanomagnetic properties of composites near the percolation threshold, the effect of a heating of the sample on the effective electrical conductivity, the concentration dependence of the effective elastic moduli, and the concentration dependence of the spectral density of the $1/f$ noise. It has also been used to describe the mechanical and electrical fracture of highly inhomogeneous composites.[4,5,7]

To describe the behavior of superconducting composites as the superconducting current rises to a level above the critical value, we use a generalization of a resistive model in which the normal current and the superconducting current flow in parallel (a resistive model for a parallel connection of two conductors: a bridge and an interlayer). We ignore the fluctuation current and the displacement current.

The effect can be outlined as follows: When the total current is above the critical level, the superconducting current and the normal current are spatially separated, flowing in different parts of the composite. In this case we ignore the Josephson properties of the bridge (whose characteristic size near the percolation threshold is much larger than ξ). The total current $I = jL^2$ (j is the current density) flowing in a characteristic volume L^3 of the composite is then given by

$$I_{c_1} + V/R + I_{c_2} \sin\varphi = I, \tag{3}$$

where V is the voltage over the length scale L, and R is the resistance of the parallel connection of bridge and interlayer [$R = R_1 R_2/(R_1 + R_2)$, where $R_1 \sim (1/\sigma_1) l/a_0^2$, and $R_2 \sim (1/\sigma_2)(a_0/s)$). Here I_{c_1} is the critical current flowing through a bridge ($I_{c_1} = j_{c_1} a_0^2$ where j_{c_1} is the critical current density of the first phase). The current I_{c_2} is the critical current of an interlayer ($I_{c_2} = j_{c_2} s$, where j_{c_2} is the critical current density of the second phase), and φ is the phase difference across the interlayer.

Interestingly, the critical current density of the composite, j_c, can be taken to be either the critical current through a bridge $(1 \gg \tau \gg (j_{c_2}/j_{c_1})^{1/(q+\nu)}$ or the critical current of an interlayer $[(j_{c_2}/j_{c_1})^{1/(q+\nu)} \gg \tau \gg (\sigma_2/\sigma_1)^{1/(t+q)}]$. Here we are ignoring the inductance and capacitance of this system.

The frequency of the Josephson emission is given by the familiar expression $\omega = 2eR\,[(I - I_{c1})^2 - I_{c_2}^2]^{1/2}$. Substituting the values of R, I_{c_1}, I_{c_2}, and I into this expression, and using (1), we find

$$\omega \sim (2e/\hbar)(a_0\tau^{-t-\nu}/\sigma_1)((j - j_{c_1}\tau^{2\nu})^2 - (j_{c_2}\tau^{-q+\nu})^2)^{1/2}. \tag{4}$$

We consider the case in which all the elements inside the blocks are superconducting, i.e., $\Delta j = j - j_{c_1}\tau^{2\nu} - j_{c_2}\tau^{-q+\nu} \ll j_{c_1}\tau^{2\nu} + j_{c_2}\tau^{-q+\nu}$. We can then write

$$\omega \sim (2e/\hbar)(a_0\tau^{-t-\nu}/\sigma_1)(\Delta j(\Delta j + 2j_{c_2}\tau^{-q+\nu}))^{1/2}. \tag{5}$$

It can be seen from (5) that for a given value of Δj (the extent to which the current density exceeds the critical value) the exponent Josephson-frequency α varies only slightly with a change in the concentration of the superconducting phase near the percolation threshold:

$$\begin{aligned} \alpha &= t + \nu \quad (\Delta j \gg 2j_{c_2}\tau^{-q+\nu}), \\ \alpha &= t + \nu/2 + q/2 \quad (\Delta j \ll 2j_{c_2}\tau^{-q+\nu}). \end{aligned} \tag{6}$$

With a further increase in the current, some of the elements inside a block go into the superconducting state, and Josephson emission appears at a different frequency.

We should also point out that Josephson junctions (interlayers) shunted externally by sufficiently small resistances (bridges) can apparently be described at a qualitative level by a resistive model.[8]

A synchronization of the Josephson generation which occurs in different parts of a sample at low currents and the critical behavior of the Josephson frequency in systems which do not have a minimum length scale will be discussed in a separate paper.

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