Aharonov–Casher oscillations in mesoscopic magnetic rings

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Some solid-state realizations of the Aharonov–Casher effect in mesoscopic magnetic rings are proposed. These realizations would be manifested by an oscillatory behavior of thermodynamic properties upon a variation of the electric field flux $\oint \vec{E} d\vec{s}$ produced by a charged filament. The oscillations would have a period hc/μ_s, where μ_s is the projection of the magnetic moment onto the filament.

Topological quantum effects which demonstrate unusual predictions of quantum physics have always attracted particular interest. The Aharonov–Bohm effect is the most famous. This fundamental quantum effect has now been tested not only in experiments with charged-particle beams in vacuum (e.g., Refs. 1 and 2) but also in condensed media.[3,4] Here it is seen as oscillations in the thermodynamic and kinetic characteristics of multiply coupled mesoscopic regions upon a variation of the magnetic flux.

In 1984, Aharonov and Casher examined a situation which was the dual of the Aharonov–Bohm effect. They showed that as a neutral particle with a magnetic moment traces out a closed loop in the electric field of a uniformly charged filament, its wave function acquires a phase shift which is independent of the shape of the trajectory:[5]

$$\Delta\phi = \frac{1}{\hbar c}\int [\vec{E},\mu] d\vec{l} = \frac{4\pi\tau\mu_s}{\hbar c} = 2\pi\frac{F}{F_0}. \tag{1}$$

Here τ is the linear charge density on the filament, μ_s is the projection of the magnetic moment onto the filament, $F = \oint \vec{E} d\vec{s} = 4\pi\tau$ is the electric field flux linking the loop trajectory, and $F_0 = hc/\mu_s$. The effect predicted in Ref. 5 was recently tested in experiments with polarized neutron beams in vacuum.[6] It is thus worthwhile to take up the question of manifestations of the Aharonov–Casher effect in condensed media.

The properties of a mesoscopic metal ring under the conditions of the Aharonov–Casher experiment were first analyzed in Ref. 7. It was shown that oscillations occur in the induced magnetization of the system as a function of the electric field flux [see (1)], with a period hc/μ_s. The experiment proposed in Ref. 7 was essentially a study of the interference of the Aharonov–Bohm and Aharonov–Casher phase shifts, since the polarization of the electrons was "implemented" by a weak magnetic field directed parallel to the charged filament. More attractive from the standpoint of a "pure" Aharonov–Casher experiment would be magnetic systems in which the electromagnetic potential interacts with only the magnetic moment of the medium. In this letter we are reporting a study of these effects.

We first consider an antiferromagnetic ring of mesoscopic size L, through whose plane a uniform charged filament with a charge density τ (per unit length) passes. At $T \neq 0$, the ring contains a finite density of thermally excited magnons. Their interaction with the electric field at the filament leads to Aharonov–Casher oscillations.

To fix the projection of the magnon spin (± 1) onto the charged filament (z), we apply a magnetic field H along the z axis. We calculate the free energy of the gas of magnons in quasi-one-dimensional antiferromagnetic chains of integer spin (Ref. 8, for example). In this case the excitation spectrum contains a gap Δ (Ref. 9), and the one-particle energies of the magnons in the external Aharonov–Casher field and the external magnetic field take the following form, in a ring of circumference L:

$$\omega_{\pm} = [(2\pi T_K)^2(n \pm F/F_0)^2 + \Delta^2]^{1/2} \pm \epsilon_H. \tag{2}$$

Here $T_K = \hbar v/L$ and $\epsilon_H = \mu H$ are characteristic energies of the quantum-size effect and of the paramagnetic splitting, v is the magnon velocity, and $\mu \simeq 2\mu_B$ is the magnetic moment of the magnons.

A standard calculation of the free-energy density of a magnon gas with spectrum (2) leads to the following exact expression for the oscillatory part of the free energy:

$$F_{osc} = \frac{4\Delta}{\pi L} \sum_{k,s=1}^{\infty} \cos(2\pi k F/F_0) \cosh(s\epsilon_H/T) \frac{K_1\{[(k\Delta/T_K)^2 + (s\Delta/T)^2]^{1/2}\}}{[k^2 + (sT_K)^2]^{1/2}}, \tag{3}$$

where $K_1(z)$ is the modified Bessel function.

According to (3), the free energy of a magnon gas in a Aharonov–Casher field oscillates as a function of the electric field flux F in (1), with a normal period hc/μ. In this sense, there is a complete analogy with the Aharonov–Bohm oscillations if we make the substitutions $F \to \Phi$ and $\mu \to e$ (Φ is the magnetic flux, and e is the electric charge). With increasing temperature, the oscillation amplitude increases. This tendency is the opposite of that observed in metals[10,3] and is analogous to the temperature dependence predicted for insulators[11] and semiconductors.[12] The reason for the anomalous temperature dependence in this case is that the equilibrium magnon density increases with increasing temperature.

At low temperatures, $T \ll \Delta$, the magnon density is exponentially small, and the Aharonov–Casher oscillations which stem from the quasiparticle contribution disappear. There is the question of the reaction of the vacuum of the antiferromagnetic ring to the Aharonov–Casher field. In the ground state, the magnetic moment of the antiferromagnet is zero, so there is no direct interaction of the electric field with the order parameter of the system (the antiferromagnetism vector). Quantum fluctuations of the spins at nodes may induce an $\vec{E} \times \vec{\mu}$ interaction in the effective long-wave Lagrangian, leading to a finite amplitude of the Aharonov–Casher oscillations even at $T=0$. We intend to analyze that problem separately; here we will analyze the response of a ferromagnetic ring in its ground state to the Aharonov–Casher field.

We recall that the nonrelativistic energy of the interaction of an electric field E with a magnetic moment μ is[13]

$$\epsilon = \frac{1}{c}\vec{\mu}[\vec{E},\vec{v}], \tag{4}$$

where $\vec{v}$ is the velocity of the particle, which carries a magnetic moment μ. For localized magnetic moments, the role of the angular velocity ($\dot{\phi}$) can be played by the generalized spin precession velocity $\dot{\Phi}$. Specifically, the energy in (4) takes the following form in the field of a charged filament (along the z axis):

$$\epsilon = \frac{2\mu_z\tau}{c}\dot{\phi} = \frac{F}{F_0}\dot{\phi} \ . \tag{5}$$

If interaction (5) is not to be a governing factor, the projection of the magnetic moment onto the filament must be conserved (and thus the z component of the resultant magnetization of the ring must be independent of the time). Furthermore, in the case of a uniform magnetization distribution (with respect to the angle ϕ),

$$M = M_0(\sin\theta\sin\Phi, \ \ \sin\theta\cos\Phi, \ \ \cos\theta), \tag{6}$$

where $\theta=\theta_0$ and $\Phi=\phi_0$, there is no reaction of the vacuum to the Aharonov–Casher field, according to (5). For topologically nontrivial configurations $\theta=\theta_0$, $\Phi=n\phi$ (it follows from the single-valuedness of M at the ring that n is an integer), we find a nonzero contribution of the Aharonov–Casher interaction to the energy of the vacuum:

$$\epsilon_{AC} = \frac{F}{F_f}\dot{\Phi}, \qquad F_f = hc/\mu_f, \tag{7}$$

where $\mu_f=\mu/n$ is a "fractional" moment at a node. It might characterize, for example, the magnetic properties of a cluster whose spins are precessing with a common phase. In other words, the quantity μ in (7) determines the magnetic moment over the magnetic coherence length.

As expected, the energy in (7) is of the form of a total time derivative, and it affects only the quantum properties of the system. It is also easy to verify that ϵ_{AC} vanishes identically in a singly connected geometry. We furthermore note that at a constant inclination $\theta=\theta_0$ this magnetization configuration of a ring with $n=1$ is exactly the same as that proposed in Ref. 14 for a study of quantum oscillations due to a Berry phase (see also Ref. 7).

The total Lagrangian of the azimuthal dynamic degree of freedom, $\Phi(t)$, incorporating the kinetic energy of the coherent precession of the spins of the ring, is the same as the Lagrangian of a quantum particle on a circle:

$$L_\Phi = \frac{I_{eff}}{2}\dot{\Phi}^2 + \frac{\Theta_v}{2\pi}\dot{\Phi}. \tag{8}$$

In our case we have $\Theta = 2\pi F/F_f$, and the effective moment of inertia depends on the anisotropic properties of the ferromagnetic particle. For an "easy-cone" anisotropy with a vertex angle θ_0, for example, we would have $I_{\rm eff} = A^{-1}(M_0/\mu)^2$, where A is the total magnetic-anisotropy energy of the ring. We know quite well that the thermodynamic characteristics of dynamic system (8) are periodic functions of the vacuum angle Θ_v. In particular, at absolute zero the oscillatory part of the ground-state energy of the ring is[15]

$$\delta E_{osc} = \frac{A}{2}\left(\frac{\mu}{M_0}\right)^2 \left\{\left\{\frac{F}{F_f}\right\}\right\}^2, \tag{9}$$

where $\{\{x\}\}$ is the fractional part of the number x, measured from the nearest integer. These oscillations are analogous to Aharonov–Bohm oscillations in quantum conductors with an incommensurate charge density wave.[15]

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