Effect of quasiparticle damping on the properties of high-T_c superconductors

V. M. Zverev and V. P. Silin
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, 117924, Moscow

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An analytic expression describing the effect of quasiparticle damping on the superconducting transition temperature is derived from the Éliashberg equations. This damping is important for determining T_c and the ratio $2\Delta_0/T_c$ for such superconductors as $Bi_2Sr_2CaCu_2O_8$ and $YBa_2Cu_3O_7$.

As far back as 1962, Culler *et al.*[1] raised the possibility that a temperature-dependent damping of quasiparticles in a superconductor might lower T_c and thereby increase the ratio $2\Delta_0/T_c$ to a value higher than the 3.53 predicted by the BCS theory. Wada[2] made an attempt to implement that idea. However, it can now be regarded as an established fact that the low T_c values of ordinary superconductors rule out any important manifestation of quasiparticle damping.

In the present letter we follow Wada[2] and derive a simple approximate expression for T_c from the Éliashberg equations.[3] This expression generalizes the McMillan–Allen–Dynes result[4,5] to include quasiparticle damping. We use experimental data on the phonon spectrum to demonstrate that damping has important effects on T_c and the ratio $2\Delta_0/T_c$ for the high-T_c superconductors.

The equations of Ref. 3, with their imaginary parts corresponding to quasiparticle damping, lead to the following equation for the real part of the superconducting gap $\Delta_1(\omega)$ near T_c

$$\Delta_1(\omega)[Z_1^2(\omega)+Z_2^2(\omega)] = Z_1(\omega)\left([\lambda-\mu^*(1+\lambda_\infty)]\int_0^{\omega_0}\frac{d\omega'}{\omega'}\mathrm{th}\left(\frac{\omega'}{2T_c}\right)\Delta_1(\omega)\right.$$
$$\left.-2\int_0^\infty d\nu\alpha^2(\nu)F(\nu)\frac{1}{\nu}\int_0^{\omega_0}d\omega'\frac{\Delta_1(\omega')}{\nu+\omega'}\right)$$
$$-2\pi\omega Z_2(\omega)\int_0^\infty d\nu\alpha^2(\nu)F(\nu)\frac{\Delta_1(\nu)}{\nu^2}\left([f(\nu)+n(\nu)][1-\frac{d\ln\Delta(\nu)}{d\ln\nu}]-\frac{df(\nu)}{d\ln\nu}\right), \tag{1}$$

where $0<\omega<\omega_0$, $Z_1(\omega)$ and $Z_2(\omega)$ are the real and imaginary parts of the renormalization function, $f(\nu)=[\exp(\nu/T_c)+1]^{-1}$, and $n(\nu)$

$= [\exp(\nu/T_c) - 1]^{-1}$. At $\omega > \omega_0$, the approximate expression $\Delta_1(\omega) = \Delta_\infty = -\mu^* \int_0^{\omega_0} (d\omega/\omega) \tanh(\omega 2T_c) \Delta_1(\omega)$ is used. Correspondingly, λ, λ_∞, μ^* have their usual form:[4–7]

$$\lambda = 2 \int_0^\infty (d\nu/\nu) \alpha^2(\nu) F(\nu), \quad \lambda_\infty = 2 \int_0^\infty (d\nu/\nu) \alpha^2(\nu) F(\nu) \ln(1 + \nu/\omega_0),$$

$$\mu^* = \mu[1 + \mu \ln(\omega_c/\omega_0)]^{-1},$$

where ω_c is the cutoff frequency ($\omega_c \gg \omega_0$).

According to Refs. 4–7, we have $Z_1 = 1 + \lambda$ at $\omega < \omega_0$. According to Ref. 2, we have $Z_2(\omega) = \Gamma(T)/\omega$, where

$$\Gamma(T) = 2\pi \int_0^\infty d\nu \alpha^2(\nu) F(\nu) [f(\nu) + n(\nu)], \tag{2}$$

is determined by the quasiparticle damping. We thus see that if we abandom the assumption $\Gamma = 0$ of Refs. 4–7, then the solution of Eq. (1) is of the form $\Delta_1(\omega) = \text{const}\,\omega^2 [\omega^2 + \Gamma^2(T_c)/(1 + \lambda)^2]^{-1}$. This frequency dependence and the incorporation of Z_2 in Eq. (1) lead to an equation for T_c which is quite different from those which have been discussed previously. Here we will write a result which corresponds to the approximation $\Gamma \ll \omega_0(1 + \lambda)$ which holds in practice, and which also corresponds to the Allen–Dynes approximation,[5] of effective phonon frequencies small in comparison with ω_0 and of a small value of μ^*:

$$T_c = T_0 \exp(-\Lambda) = 1,134 \omega_{\ln} \exp\left(-\frac{1 + \lambda}{\lambda - \mu^*} - \Lambda\right). \tag{3}$$

Here T_0 corresponds to the approximation in which the quasiparticle damping is ignored, $\omega_{\ln}$ is given by the usual expression[5,8]

$$\omega_{\ln} = T_c \exp[(2/\lambda) \int_0^\infty (d\nu/\nu) \alpha^2(\nu) F(\nu) \ln(\nu/T_c)],$$

and Λ determines the change caused in T_c by the quasiparticle damping. It is given by

$$\Lambda = \frac{\lambda A_1 + \lambda_2 - \delta\lambda_0}{\lambda - \mu^*}, \text{ where } A_1(T_c) = \gamma^2 \int_0^\infty \frac{d\omega}{\omega} \frac{\text{th}(\omega/2T_c)}{\omega^2 + \gamma^2},$$

$$\lambda_2(T_c) = 2\pi\gamma \int_0^\infty d\nu \frac{\alpha^2(\nu) F(\nu)}{\nu^2 + \gamma^2} \left([f(\nu) + n(\nu)][1 - \frac{d}{d \ln \nu} \ln \frac{\nu^2}{\nu^2 + \gamma^2}] - \frac{df(\nu)}{d \ln \nu} \right),$$

$$\delta\lambda_0(T_c) = 2\gamma^2 \int_0^\infty \frac{d\nu}{\nu} \frac{\alpha^2(\nu) F(\nu)}{\nu^2 + \gamma^2} (\ln \frac{\gamma}{\nu} + \frac{\pi\nu}{2\gamma}), \quad \gamma = \frac{\Gamma(T_c)}{1 + \lambda}.$$

For the superconducting gap Δ_0 at $T=0$ we use the solution of the Éliashberg equations from Ref. 9:

$$\Delta_0 = 2\omega_0 \exp\left(-\frac{1+\lambda+\lambda_0-5\chi}{\lambda-\mu^*(1+\lambda_\infty)}\right),$$

where

$$\lambda_0 = 2\int_0^\infty \frac{d\nu}{\nu}\alpha^2(\nu)F(\nu)\ln(1+\frac{\omega_0}{\nu}), \quad \chi = \int_{\Delta_0}^\infty d\nu\alpha^2(\nu)F(\nu)\frac{\Delta_0^2}{\nu^3}\ln\frac{\nu}{\Delta_0}.$$

In the Allen–Dynes limit we then find

$$\Delta_0 = 2\omega_{\ln}\exp\left(-\frac{1+\lambda-5\chi}{\lambda-\mu^*}\right). \tag{4}$$

Like (3), the last expression is independent of the particular choice of the parameter ω_0.

From expressions (3) and (4) we find

$$\frac{2\Delta_0}{T_c} = 3.53\exp\left(\frac{5\chi}{\lambda-\mu^*}+\Lambda\right). \tag{5}$$

The first term in parentheses here is determined by the strong-coupling effect.[9] The second is due to quasiparticle damping. For conventional superconductors with $T_c \leqslant 10$ K, quasiparticle damping is extremely unimportant, since we have $\Lambda \leqslant 1\%$.

For the high-T_c superconductors, in contrast, the effect of quasiparticle damping turns out to be extremely important, because of the high value of T_c. We first consider $Bi_2Sr_2CaCu_2O_8$, for which we will use the results for α^2F from Ref. 10. From those results, with $\lambda = 4$ and $\mu^* = 0$ (or 0.1), we find the following results from the expressions above: $\gamma = 3.4$ meV, $\omega_{\ln} = 28.6$ meV, $T_0 = 108$ K (or 104 K), $T_c = 82$ K (or 80 K), and $\Delta_0 = 22$ meV (or 21 meV). The experimental values are[10] $T_c = 82$–87 K and $\Delta_0 = 20$ meV. The corresonding calculated value is $2\Delta_0/T_c = 6.2$. In this case we have $\Lambda = 0.27$ and $5\chi/(\lambda-\mu^*) = 0.29$ (or 0.30), so we can say that quasiparticle damping has an effect on the ratio $2\Delta_0/T_c$ which is comparable to that of the strong coupling.

We turn now to $YBa_2Cu_3O_7$, for which we have data on only $F(\omega)$, from neutron-scattering experiments (Ref. 11). Without making any claim that the absolute values are correct in the assumption (which lacks a solid basis) that α^2 is independent of ω, we conclude that those data lead to the following results according to the expressions presented above, with $\lambda = 4$ and $\mu^* = 0$ (or 0.1): $\gamma = 6.2$ meV, $\omega_{\ln} = 23.5$ meV, $T_0 = 89$ K (or 86 K), $T_c = 66$ K (or 64 K), and $\Delta_0 = 17$ meV (or 16 meV). The experimental values,[12] on the other hand, are $T_c = 90$ K and $\Delta_0 = 19$ meV. We find the corresponding calculated value $2\Delta_0/T_c = 5.9$. Here we have $\Lambda = 0.30$ and $5\chi/(\lambda-\mu^*) = 0.22$. Improvements in the accuracy of the experimental determination of

$F(\omega)$ in the high-frequency region might change the value of $\omega_{\ln}$, so the value of the constant λ, required for this interpretation, may also change.

In summary, we have found that quasiparticle damping has an important effect on the properties of the high-T_c superconductors. This conclusion has something in common with a result found by Allen and Rainer.[13] Their numerical calculations demonstrated that quasiparticle damping would have an important manifestations in the nuclear-spin relaxation rate in a high-T_c superconductor.

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