Turning a beam of high-energy charged particles by means of scattering by atomic rows of a curved crystal

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A stable turning of negatively and positively charged high-energy particles by means of multiple scattering by the atomic rows of a curved crystal is predicted. This effect could be observed at existing accelerators.

1. At high energies the motion of a charged particle along a crystallographic plane in a crystal is determined primarily by the continuous potential of the plane, i.e., by the lattice potential averaged over the coordinates of the planes along which the motion occurs.[1] If the motion occurs in a curved crystal, a beam of positively charged particles could be turned through an angle significantly larger than the critical angle for planar channeling.[2] Turning occurs for particles which are executing a finite motion in the continuous potential of the crystallographic planes of a curved crystal.

In the present letter it is shown that a significant turning of a beam of high-energy charged particles can be achieved not only in the case of motion along crystallographic planes but also in the case in which the particles are moving along a crystallographic axis in a curved crystal. A distinctive feature of this effect is that the beam is turned not as the result of finite motion in the field of the atomic rows but as a result of particular features of the multiple scattering of above-barrier particles by the atomic rows of the curved crystal. This effect has the important advantage that in the case of scattering by atomic rows, beams of either positively or negatively charged particles can be turned.

2. As a fast charged particle strikes a crystal at a small angle ψ_0 from one of the crystallographic axes (the z axis), it undergoes successive collisions with various atomic rows which are parallel to this axis. The scattering which occurs in the collision with each row is primarily along the azimuthal angle φ, in the plane transverse with respect to the axis of the row.[3] The multiple scattering by the atomic rows results in a redistribution of the particles with respect to the angle φ. If the collisions of the particle with the various rows can be treated as random (this assumption is valid if the conditions for a dynamic chaos hold during the motion of the particle in the crystal[4]), and if the angle (ψ) between the momentum of the particle and the z axis is sufficiently small, the φ distribution of the particles becomes uniform extremely rapidly. As a result, the center of the scattered beam is directed along the crystallographic axis. In other words, as the beam is scattered by the atomic rows of the crystal, its axis is rotated through an angle $\psi = \psi_0$.

3. In a curved crystal, the particles undergo a redistribution with respect to the azimuthal angle with respect to the instantaneous direction of the crystallographic axis

(φ) and also with respect to the polar angle from this axis. The center of the beam is displaced in the direction of the bending of the crystal in the process. We wish to call attention to certain distinctive features of the scattering of fast particles by a crystal under these conditions.

For this purpose we consider the very simple case in which the angle (ψ) between the beam direction and the row axis undergoes only a small change over the duration of the interaction of the particle with an individual curved row, $\tau \sim \min(2a_{TF}/\psi, 2a_{TF}/\psi_c)$, where a_{TF} is the Thomas-Fermi radius of the atom, and ψ_c is the critical angle for axial channeling. In this case the scattering by each atomic row can be treated in the approximation $\psi = \text{const}$. As we go from row to row, the angular coordinates of the particle thus vary in accordance with the recurrence relations

$$\begin{aligned} \theta_{x,i+1} &= \theta_{x,i}\cos\varphi_i + \theta_{y,i}\sin\varphi_i, \\ \theta_{y,i+1} &= \theta_{y,i}\cos\varphi_i - \theta_{x,i}\sin\varphi_i - \frac{\tau_i V_{11i}}{R}, \\ \psi_{i+1} &= (\theta_{x,i+1}^2 + \theta_{y,i+1}^2)^{1/2}, \end{aligned} \tag{1}$$

where $\theta_{x,i+1}$ and $\theta_{y,i+1}$ are angular coordinates with respect to the instantaneous direction of the crystallographic axis after collision i with the row, τ_i is the time between successive collisions, R is the radius of curvature of the crystal, and $\varphi_i = \varphi_i(\varphi_i, b_i)$ is the azimuthal scattering angle in collision i with the row. This angle depends on the polar angle ψ_i and on the impact parameter of the row, b_i.

Relations (1) are convenient for studying the particle dynamics in a curved crystal and for numerical simulations of the passage of particles through crystals in this case. We note in this connection that the motion of fast particles in the periodic field of atomic row of a crystal may be either regular or random. In general, therefore, the picture of the passage of these particles through the crystal is rather complex and requires a numerical simulation with allowance fo the actual geometry. Below we report some results of a simulation of the passage of fast, negatively and positively charged particles through a curved crystal in the random-row approximation. This approximation simplifies the calculations substantially and speeds them up. This approximation is valid if the motion of the overwhelming majority of the particles in the periodic field of the atomic rows is random. This requirement is satisfied, in particular, over a extremely broad range of the angle ψ ($\psi \leqslant 3\text{–}5\psi_c$), i.e., the angle between the momentum of the particle and the crystallographic axis, as fast, negatively charged particles pass through a crystal.[4] We might add that the random-row approximation makes it a fairly simple matter to estimate the effect of incoherent scattering on the motion of the beam particles.

4. Figure 1 shows the results of a simulation of the passage of negatively charged particles with an energy $E = 100$ GeV through a curved silicon crystal, along the $\langle 111 \rangle$ axis. The quantities plotted along the ordinate and abscissa are the angular coordinates of the scattering of the particles by the curved crystal. The circles correspond to the initial and final directions of the crystallographic axis; a heavy point corresponds to the initial direction of the particle beam; and the other points show the angular coordinates of the beam particles as they leave the crystal.

These results show that at $\psi \leqslant 0.5\psi_c$, and for a crystal with a large radius of curvature, essentially all the particles of the incident beam follow the crystallographic axis of the curved crystal (Fig. 1a). The reason is that in this case those particles which are moving at an angle $\psi \leqslant \psi_c$ from the crystallographic axis are able, by virtue of multiple scattering by the atomic rows, to enter that interval of the azimuthal angle φ in which the angle ψ (the polar angle of the velocity of the particle with respect to the instantaneous direction of the atomic row) decreases because of the curvature of the crystal. As a result, such particles follow the direction of the crystallographic axis. We wish to stress that the turning of the beam in this case occurs for above-barrier particles.

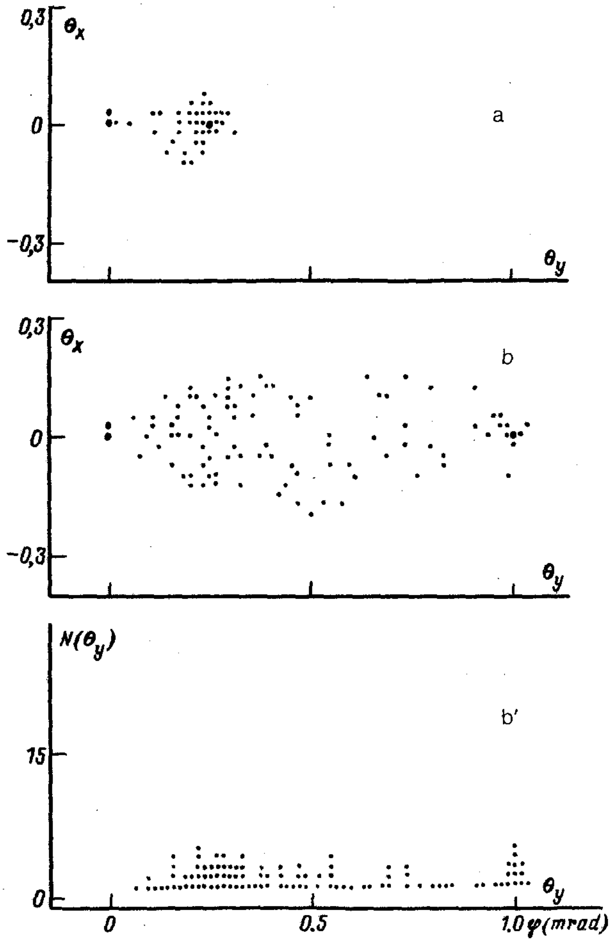

FIG. 1. Results of a numerical simulation of the scattering of a beam of negatively charged particles incident at an angle $\psi_0 = 0.5\psi_c$ on the $\langle 111 \rangle$ axis of a curved Si crystal. The energy of the beam particles is $E = 100$ GeV. The radius of curvature of the crystal is $R = 40$ m. The thickness of the crystal is (a) 1 cm or (b) 4 cm. The statistical base is $N = 100$ particles. b′—Histogram for case *b*.

For positively charged particles, the efficiency with which the beam follows the crystallographic axis is greater in this model (Fig. 2). However, for positively charged particles some of the beam particles may be captured into a regime of stable motion along crystallographic planes (planar channeling), even if $\psi < \psi_c$. Consequently, in describing the motion of such particles it becomes necessary to deal with the periodic nature of the atomic rows in the plane orthogonal with respect to the crystallogrpahic axis. That refinement goes beyond the scope of the present letter.

With an increase in the crystal thickness (Fig. 1b), the beam broadens, and its center shifts with respect to the final direction of the axis. Reducing the radius of curvature of the crystal and incorporating incoherent scattering in the simulation lead to the same effects. As the energy is raised, however, the relative effect of the latter process on the scattering decreases rapidly, and at teraelectron-volt energies the effect of the incoherent scattering on the motion of the particles along the crystallographic

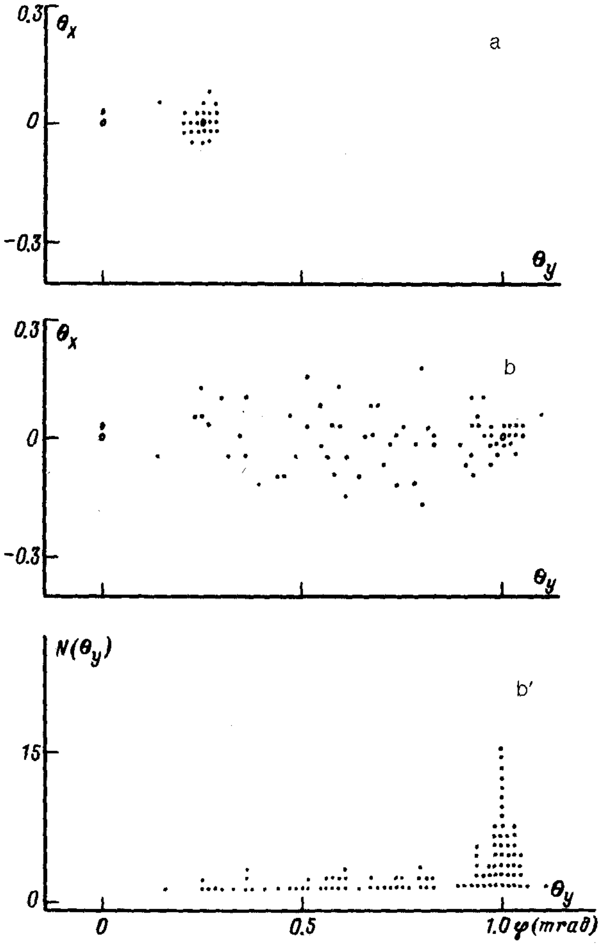

FIG. 2. The same as in Fig. 1, but for positively charged particles.

axes of the curved crystal can be ignored. We might note in this connection that, as the energy (E) of the particle beam is varied, recurrence relations (1) are numerically invariant under the following transformations of parameters:

$$l = l_0(E/E_0)^{1/2}; \quad R = R_0E/E_0; \quad \theta_i = \theta_{0i}(E_0/E)^{1/2}, \tag{2}$$

where l_0 and R_0 are the thickness and radius of curvature of the crystal, and θ_{0i} are the angular coordinates of the particles at $E_0 = 100$ GeV. It follows in particular that the results in Figs. 1 and 2 could also be used to study the passage of particles with a different energy through a curved crystal.

The experimental data presently available[5] agree with these results. The conditions under which these experiments were carried out ($E \sim 10$ GeV, $R \sim 2$ m), however, were such that it was not possible to see the stable tracking of an axis of the curved crystal by the beam which we mentioned earlier. The latter effect should be seen at high particle energies and at large radii of curvature of the crystal. We believe that an experimental test of this effect would be of substantial interest, since the idea of spilling teraelectron-volt particle beams from the UNK and SSC colliders is presently being discussed in several laboratories.[6] Achieving this spilling would require the stable turning of some of the particles from the collider through an angle on the order of 10^{-4} rad. The results found above show that an effective turning of the beam through such an angle might be feasible in the case of the scattering of the particles by the atomic rows of a curved crystal. The effect could be tested, according to (2), at existing accelerators, at energies on the order of 100 GeV.

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