Polarization of incoherent emission by relativistic electrons and positrons in a crystal

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A polarization is predicted in the incoherent emission of relativistic particles in a crystal. The effect depends on the sign of the charge of the particle. The magnitude of the effect increases rapidly with decreasing angle between the particle's momentum and the crystallographic plane. This effect could be observed experimentally.

The cross section for radiation by relativistic electrons in an oriented crystal can be written as the sum of the cross sections for coherent and incoherent emission.[1] This result is found in a first-order perturbation theory in the interaction of the particle with the lattice field. Analysis of corrections to the cross section for coherent emission has shown[2] that these corrections increase rapidly with decreasing value of θ, the angle of incidence of the particle on the crystal, reckoned from the crystallographic planes.

In the present letter we derive a correction to the cross section for the incoherent emission of relativistic particles in an oriented crystal. This correction reflects the part of the emission which corresponds to the next higher order of the perturbation theory in the interaction of the particle with the crystal field. We will see that this correction makes a governing contribution to the polarization of the incoherent emission, that the polarization direction depends on the sign of the particle's charge, and that the effect increases rapidly with decreasing angle θ. If this correction is not made, the polarization of the incoherent emission is zero.[1] This effect might be utilized to produce beams of polarized γ rays with energies close to that of the radiating particle.

The process by which a relativistic electron emits radiation in a medium develops in a large spatial region along the particle's momentum. This region grows rapidly with the energy.[1] If the electron collides with a large number of crystal atoms in this region, the effective constant of the interaction of the electron with these lattice atoms may be large in comparison with unity, so we could use the semiclassical approximation to describe the emission of radiation by the electron in the crystal.[3] In the dipole approximation the cross section for the emission of a photon with an energy ω, a momentum $\vec{k}$, and a polarization $\vec{e}$ is

$$d\sigma = \frac{e^2\omega d\omega}{4\pi^2}\frac{\epsilon}{\epsilon'}\int \frac{d^2\rho_0 do}{(\omega - k\cdot v)^2}\left\{\left|\vec{e}\cdot\vec{W} + \vec{e}\cdot\vec{v}\frac{\vec{k}\cdot\vec{W}}{\omega - kv}\right|^2 + \vec{e}^{\,2}|\vec{W}|^2\frac{\omega^2}{4\epsilon\epsilon'}\right\}, \qquad (1)$$

where do is an element of solid angle along the $\vec{k}$ direction, $\vec{v}$ is the initial velocity of the electron, ϵ is its energy, $\epsilon' = \epsilon - \omega$, $\vec{W} = \int_{-\infty}^{\infty} dt \dot{\vec{v}}_{\perp}(t) \exp[i(\epsilon/\epsilon')(\omega - \vec{k}\cdot\vec{v})t]$ is the component of the particle's velocity which is orthogonal to $\mathbf{v}$ in the crystal, and ρ_0 is the impact parameter.

Let us examine the linear polarization of the radiation as the electron moves through a crystal at a small angle θ from one of the crystallographic planes [the y,z plane]. This polarization is given by[1]

$$P = \frac{d\sigma_1 - d\sigma_2}{d\sigma_1 + d\sigma_2}, \qquad (2)$$

where $d\sigma_1$ and $d\sigma_2$ are the cross sections for the emission of photons with polarization vectors $\vec{e}_1 = \vec{k} \times \vec{e}_x / |\vec{k} \times \vec{e}_x|$ and $\vec{e}_2 = \vec{k} \times \vec{e}_1/\omega$, and $\vec{e}_x$ is the unit vector along the x axis. The latter is orthogonal to the y,z plane. Since the angles at which the relativistic particle radiates are small, one can easily show that

$$P = \frac{1}{2}\frac{|W_y|^2 - |W_x|^2}{|W_y|^2 + |W_x|^2}, \qquad (3)$$

where $W_{y,x}$ are components of the vector $\vec{W}$ along the y and x axes.

In the case at hand, in which the radiation formation length $l_c = 2\epsilon\epsilon'/m^2\omega$ is large in comparison with the lattice constant a, we can assume, in the calculation of $\vec{W}$, that the velocity of the electron changes abruptly in the collision with each atom. We can thus write

$$\vec{W} \approx \sum_n \vec{\vartheta}_n \exp\left[i\frac{\epsilon}{\epsilon'}(\omega - \vec{k}\cdot\vec{v})t_n\right], \qquad (4)$$

where $\vec{\vartheta}_n$ is the scattering angle in the collision with atom n, and t_n is the time at which the collision occurs.

Using the transformations used in Ref. 4, and taking an average over the thermal vibrations of the atoms in the lattice, we find the following expressions for the quantities in (3):

$$\int d^2\rho_0 |W_{x,y}|^2 = F_{x,y}^{(n)} + F_{x,y}^{(\mathrm{coh})}, \qquad (5)$$

where $F_{x,y}^{(n)}$ and $F_{x,y}^{(\mathrm{coh})}$ are the incoherent and coherent components of the quantity of interest. We will be interested below in the radiation at high frequencies, at which the term $F^{(n)}$ is predominant in the radiation. [The term $F^{(\mathrm{coh})}$ in (5) can be ignored if $l_c \ll a/\theta$.] This predominant term is given by

$$F_{x,y}^{(n)} = \frac{1}{16\pi^4\epsilon^2}\frac{1}{a_y a_z}\int dy dt \int d^2g d^2g' \, \vec{g}_{x,y} \cdot \vec{g}_{x,y}' U(\vec{g}) U(\vec{g}')$$
$$\times [e^{-\frac{1}{2}\overline{u^2}(\vec{g}-\vec{g}')^2} - e^{-\frac{1}{2}\overline{u^2}(\vec{g}^2+\vec{g}'^2)}] e^{-i(\vec{g}-\vec{g}')\cdot\vec{\rho}(t)}, \qquad (6)$$

where a_y and a_z are the lattice constants along the y and z axes, $U(g)$ is the Fourier component of the potential of an individual lattice atom in the case $g_z = 0$, u^2 is the mean square magnitude of the thermal vibrations of the atoms in the crystal, and $\rho(t)$ is the trajectory of the particle in the continuous potential $U_p(x)$ of the y,z crystallographic planes.

The trajectory $\vec{\rho}(t)$ in (6) is determined by the equation[5]

$$\epsilon\ddot{\vec{\rho}} = -\vec{\nabla} U_p(x). \tag{7}$$

Under the condition $\epsilon_\perp = \frac{1}{2}\epsilon\theta^2 \gg |U_p|$, the trajectory of a particle in the potential $U_p(x)$ is approximately rectilinear. Retaining the first term in the expansion of the trajectory in the potential $U_p(x)$, we find

$$\vec{\rho}(t) = y\vec{e}_y + \theta t\vec{e}_x - \vec{e}_x \frac{1}{\epsilon} \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \frac{\partial}{\partial x} U_p(x)|_{x=\theta t''}, \tag{8}$$

where y is the impact parameter, and $\vec{e}_y$ and $\vec{e}_x$ are unit vectors along the y and x axes.

Substituting (8) into (6), we find the polarization of the radiation, with allowance for the slight deviation of the trajectory from rectilinearity in the potential $U_p(x)$. If we ignore the terms proportional to $U_p(x)$ in (6), we find the known result[1] $P=0$.

Singling out the dimensional quantities in (2), we find, in the first approximation in $U_p(x)$,

$$P = \pm\frac{\theta_p^2}{\theta^2}\eta, \tag{9}$$

where the $+$ and $-$ refer to positrons and electrons, $\theta_p = \sqrt{2|U_p|_{\max}/\epsilon}$ is the critical angle for planar channeling,[5] and η is a numerical factor on the order of unity. The integral over g in (6) diverges at large values of g. The divergence results from the use of the dipole approximation, which is valid at $g \lesssim mc$, to describe the radiation. Taking this constraint into account, we find $\eta \approx 1/3$ with logarithmic accuracy for a screened Coulomb potential.

Correcting the cross section for the incoherent emission of relativistic particles in a crystal thus leads to a nonzero linear polarization of the radiation. This polarization arises from the asymmetry of the motion of a particle in the continuous potential of the crystallographic planes, $U_p(x)$.

Expression (9) shows that the polarization depends on the sign of the charge of the particle. For electrons the polarization vector lies in the y,z plane, while for positrons it is perpendicular to this plane. With decreasing θ, the polarization P increases rapidly. The curvature of the particle's trajectory in the potential $U_p(x)$ must be taken into account more carefully at $\theta \sim \theta_p$.

The estimates above show that the magnitude of this effect would be quite sufficient to be observed experimentally. The effect could be seen at those energies of the emitting particles and at those frequencies of the emitted photons for which the term describing the incoherent effect dominates the emission.

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