Chiral superparticle and closure of the Siegel *k*-symmetry algebra in a Lagrangian formulation

A. A. Deriglazov
Tomsk Polytechnical Institute, 634028, Tomsk

(Submitted 17 September 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **54**, No. 9, 533-535 (10 November 1991)

An action is proposed for a model of a chiral superparticle. This action includes a set of auxiliary fields with a closed algebra of local symmetries. A partial fixing of the gauge imposed on the auxiliary field generates a local Siegel k symmetry.

Several ways to close the algebra of the local Siegel k symmetry[4] for a Brink-Schwarz superparticle in $d=10$ (Ref. 5) have been proposed.[1-3] These ideas involve introducing a set of auxiliary fields in a Hamiltonian formalism. No Lagrangian formulation has been devised, nor has a corresponding closure mechanism been devised for the Green–Schwarz superstring.[6] Other versions of the closure of the algebra of the local Siegel k symmetry (LSKS) may prove useful here. In the present letter we analyze, at a classical level, the action for a chiral superparticle in $d=2$ in two forms, differing in the particular choice of auxiliary fields. The first action is invariant under the LSKS, and the second under the local-symmetry group with a closed algebra. A partial fixing of the gauge imposed on the auxiliary field in the second of these actions leads to the first, generating an LSKS: The remaining gauge transformations, which do not disrupt the selected gauge, are the LSKS.

An action for a chiral particle (i.e., for a theory with a Lagrangian $L \propto \frac{1}{e}\dot{x}^{+}\dot{x}^{-}$ and the auxiliary condition $\dot{x}^{-}=0$ or with a constraint $p^{-}\approx 0$ in the Hamiltonian formalism) was proposed by Gomes *et al.*[7] in a form which is not explicitly Lorentz-invariant:

$$S=\int d\tau \frac{1}{e}\dot{x}^{1}(\dot{x}^{0}-\dot{x}^{1}). \tag{1}$$

A Lorentz-invariant action with the auxiliary field $\lambda^{++}(\tau)$ (an analogy of the Siegel action for a chiral boson[8]) leads to the same Lagrange equations as in (1):

$$S=\int_{0}^{T} d\tau\left[\frac{1}{e}\dot{x}^{+}\dot{x}^{-}-\lambda^{++}(\dot{x}^{-})^{2}\right]. \tag{2}$$

The field λ^{++} drops out of the equations of motion: $(\frac{1}{e}\dot{x}^{+})^{\cdot}=(\dot{x}^{-})^{2}=0$. In a Hamiltonian formalism, a theory with two primary constraints of the first kind corresponds to action (2), indicating that in addition to the reparametrization invariance with the parameter $\alpha(\tau)$,

$$\delta_{\alpha}x^{\pm}=\alpha\dot{x}^{\pm},\ \ \delta_{\alpha}e=(\alpha e)^{\cdot},\ \ \delta_{\alpha}\lambda^{++}=\alpha\dot{\lambda}^{++}-\dot{\alpha}\lambda^{++}, \tag{3}$$

there is yet another local symmetry. Specifically, under transformations with the parameter $B^{++}(\tau)$,

$$\delta_B x^+ = eB^{++}\dot{x}^-, \quad \delta_B \lambda^{++} = \frac{1}{2}\dot{B}^{++} + \frac{\dot{e}}{e}B^{++}, \tag{4}$$

a variation of action (2) leads to $\delta L = [\frac{1}{2}B^{++}(\dot{x}^-)^2$. We thus have invariance under the following auxiliary boundary condition on B^{++}: $B^{++}(0)[\dot{x}^-(0)]^2 = B^{++}(T)[\dot{x}^-(T)^{\cdot}]^2$. Transformations (4) make it possible to assign a gauge to the field λ^{++}; the gauge $\lambda^{++}=0$ fixes symmetry (4) completely. We also note that the action for chiral particle can be rewritten in a form with two auxiliary fields, λ^{++} and ω^+:

$$S = \int d\tau \left[\frac{1}{e}\dot{x}^+\dot{x}^- - \lambda^{++}(\dot{x}^-)^2 - \omega^+\dot{x}^-\right]. \tag{5}$$

In addition to (3) and (4), there is the trivial local symmetry with the parameter $\xi^+(\tau)$:

$$\delta x^+ = \xi^+, \quad \delta\omega^+ = \frac{1}{e}\dot{\xi}^+. \tag{6}$$

Let us examine the supersymmetric analogies for (2) and (5) which are found by making the substitution $\dot{x}^\mu \to \Pi^\mu \equiv \dot{x}^\mu - i\bar{\theta}\sigma^\mu\dot{\theta}$, where $\theta_\alpha = (\theta_1,\theta_2)$ is a Majorana spinor. Choosing a representation for the matrices σ^μ in the form

$$\sigma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad C^{\alpha\beta} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \bar{\theta} = C\theta, \tag{7}$$

we find $\Pi^+ = \dot{x}^+ - i\theta_1\dot{\theta}_1$ and $\Pi^- = \dot{x}^- - i\theta_2\dot{\theta}_2$. For (2) we find the action of the chiral superparticle,

$$S = \int d\tau \left[\frac{1}{e}\Pi^+\Pi^- - \lambda^{++}(\Pi^-)^2\right], \tag{8}$$

which has the local Siegel k symmetry (in addition to the reparametrizations):

$$\delta_k\theta_1 = \Pi^+ k, \quad \delta_k x^+ = (\theta_1 k)\Pi^+, \quad \delta_k e = 2ie(\dot{\theta}_1 k). \tag{9}$$

The algebra of these transformations is closed on the fields x^+, θ_1: $[\delta_k,\delta_{k'}] = \delta_\alpha + \delta_{\tilde{k}}$ $(\alpha_+ 2ikk'\Pi^+$, $\tilde{k} = 2i\dot{\theta}_1 kk'$. It is open for the field e $[\delta_k,\delta_{k'}]e = \delta_{\tilde{k}}e + \delta_\alpha e + 2ikk'(\dot{\Pi}^+ e - \Pi^+\dot{e})$.

The supersymmetric analog of (5),

$$S = \int d\tau \left[\frac{1}{e}\Pi^+\Pi^- - \lambda^{++}(\Pi^-)^2 - \omega^+\Pi^-\right], \tag{10}$$

has the following group of local symmetries with the Bose parameters $\alpha(\tau)$, $\xi^+(\tau)$, and $b(\tau)$ and the Fermi parameter $\epsilon(\tau)$:

$$\begin{aligned}\delta_\alpha x^\pm = \alpha \dot{x}^\pm, \quad & \delta_\alpha \lambda^{++} = -\dot{\alpha}\lambda^{++} + \alpha\dot{\lambda}^{++}, \\ \delta_\alpha \theta = \alpha\dot{\theta}, \quad & \delta_\alpha \omega^+ = -\dot{\alpha}\omega^+ + \alpha\dot{\omega}^+, \\ & \delta_\alpha e = (\alpha e)^{\cdot}.\end{aligned} \tag{11}$$

$$\delta_\xi x^+ = \xi^+, \ \delta_\xi \omega^+ = \frac{1}{e}\dot{\xi}^+. \tag{12}$$

$$\begin{aligned}&\delta_\epsilon \theta_1 = \epsilon, \ \delta_\epsilon x^+ = -i\theta_1\epsilon, \\ &\quad \delta_\epsilon \omega^+ = -\frac{2i}{e}\theta_1\dot{\epsilon}.\end{aligned} \tag{13}$$

$$\delta_b \omega^+ = \frac{b}{e}\Pi^+, \quad \delta_b e \doteq -be. \tag{14}$$

[The λ^{++}, ω^+ transformation laws in the course of reparametrizations can be put in the standard form by multiplying the second and third terms in (10) by $1/e$. The form given is convenient for the discussion below.] A direct calculation verfies that the complete algebra, (11)–(14), is closed.

After the partial gauge $\omega^+ = 0$ is imposed, action (10) becomes (8) (in a Hamiltonian formalism, the constraint $p_\omega \approx 0$ does not generate any independent secondary constraints), so we would naturally expect that residual gauge transformations for (10) which do not disrupt the selected gauge coincide with the Siegel k symmetry. The change in ω^+ under (12)–(14) can be written in the form

$$\delta\omega^+ = \frac{1}{e}[\xi^+ - 2i\theta_1\epsilon]^{\cdot} \ + \frac{1}{e}[2i\dot{\theta}_1\epsilon + \Pi^+ b], \tag{15}$$

from which we see that the condition $\delta\omega^+ = 0$ holds if $\epsilon \sim \Pi^+$ and $b \sim \dot{\theta}_1$. Choosing $\epsilon = \Pi^+ k$, where k is a new independent parameter, we obtain $b = -2i\theta_1 k$, $\xi^+ = 2i\theta_1\epsilon$. Using the notation $\delta_\epsilon + \delta_\epsilon + \delta_b = \delta_k$, we find that the transformation laws for the fields x^+, θ_1, and e under the residual gauge transformations δ_k coincide with (9). The algebra obtained as a result is open, since the initial parameters of the transformations, ϵ, ξ, and b, were chosen to be field-dependent in the transition to the parameter k. Furthermore, transformations (13) with $\epsilon = \text{const}$ do not disrupt the gauge; they generate a global supersymmetry for (8).

By setting $\lambda^{++} = 0$ in (10) and going through all the calculations again, we would obtain a theory with a Lagrangian $L = \frac{1}{e}\Pi^+\Pi^-$ and the auxiliary condition $\Pi^- = 0$.

I am indebted to S. M. Kuzenko for useful discussions.

[1]J. Kowalski-Glikman and J. W. van Holten, "On the quantum theory of the superparticle," Preprint NIKHEF-H/88-16, Amsterdam, 1988.
[2]D. P. Sorokin, V. I. Tkach, and D. V. Volkov, Mod. Phys. Lett. A 4, 901 (1989).

[3]E. Sokatchev, Class. Quant. Grav. **4**, 237 (1987).
[4]W. Siegel, Phys. Lett. B **128**, 397 (1983).
[5]L. Brink and H. Schwarz, Phys. Lett. B **100**, 310 (1981).
[6]M. Green and J. Schwarz, Phys. Lett. B **136**, 367 (1984).
[7]M. Gomes, V. O. Rivelles, and A. J. Da Silva, Phys. Lett. B **218**, 63 (1989).
[8]W. Siegel, Nucl. Phys. B **238**, 307 (1989).

Translated by D. Parsons