Manifestations of quantum fluctuations in optical solitons: comment on a previous letter

A. V. Belinskiĭ

M. V. Lomonosov Moscow State University, 119899, Moscow

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The shape of a fundamental Schrödinger soliton does not change as the soliton propagates through an intrinsically nonlinear optical fiber. The broadening of the expectation value of the envelope is due exclusively to an increase in the quantum uncertainty regarding the position of the crest of the envelope.

In Ref. 1 the exact soliton-like solution[2] of the quantum-mechanical Schrödinger equation was used to analyze the evolution of the second moment of the field, $\langle N(x,t)\rangle \equiv \langle \phi^{+}(x,t)\phi(x,t)\rangle$, during the propagation of a soliton through an ideally transparent fiber. Here x and t are the normalized coordinate and the normalized propagation time, and ϕ^{+} and ϕ are normalized operators representing the negative- and positive-frequency parts of the field. The integral $\int \langle N(x,t)\rangle dx = n_0$ is the expectation value of the number of photons in the soliton. It was established that with increasing time t, i.e., with increasing distance traversed, the expectation value of the envelope, $\langle N(x,t)\rangle$, spreads out. It was concluded on this basis that the soliton suffers a gradual degradation.

However, a broadening of $\langle N(x,t)\rangle$ might be caused by two factors: (a) a spreading of the soliton or (b) a manifestation of an uncertainty regarding the position of the soliton, since the expectation value is calculated over the ensemble of solitons. The contribution of each of these two factors can be determined by evaluating the intensity correlation function

$$K(x',t) = \int < N(x,t)N(x+x',t) > dx, \tag{1}$$

which does not depend on the coordinate x (or on the uncertainty regarding this coordinate).

To calculate this function, in the general case, is rather difficult. In the particular case $x' = 0$, on the other hand, one can show that, under the condition $n_0 \gg 1$, we have

$$K(x'=0,t) = n_0|C|(n_0^2 + 3n_0 - 1)/6 + n_0 \simeq n_0^3|C|/6, \tag{2}$$

where C is a nonlinearity parameter.

We thus conclude that $K(0)$ does not evolve as time elapses. Interestingly, the integral $\int |\phi_0(x,t)|^4 dx$ is also equal to $n_0^3|C|/6$, where $\phi_0(x,t)$ is a C-number solution of the classical Schrödinger equation (Ref. 3, for example).

From (1) we find that

$$\int K(x')dx' = < [\int N(x,t)dx]^2 > \equiv n_0^2 + n_0 \simeq n_0^2 \quad (3)$$

is also independent of the time. Consequently, the width of the correlation function, $\bar{x} \simeq 6/n_0|C|$, also remains constant, telling us that the soliton does not spread out. Here we are assuming that there are no side peaks in $K(x')$, since they could mean only a "collapse" of the soliton into localized groups of photons. Since $K(0)$ does not change, the peak intensities of these groups should be greater than the initial intensity at the crest of the soliton, $\langle N(0,0) \rangle$. Such a situation seems improbable.

The results presented here are supported by the following considerations. The quantum-mechanical uncertainty regarding the coordinate of the soliton, Δx, can be calculated approximately by taking the approach of Ref. 3:

$$\Delta x(t) = (\pi^2/n_0^3 C^2 + n_0 C^2 t^2)/3 \simeq n_0 C^2 t^2/3, \quad n_0 \gg 1 \ . \quad (4)$$

The characteristic time (t_x) over which a random displacement Δx reaches the half-width of the soliton is thus

$$t_x \simeq T\sqrt{n_0}/4, \quad (5)$$

where $T = 8\pi/n_0^2 C^2$ is the period of the soliton.

On the other hand, the uncertainty in the momentum is

$$\Delta p = |C|\sqrt{n_0/12} \gg |C|, \quad (6)$$

again under the assumption $n_0 \gg 1$. According to Refs. 1 and 2, the time scale for a doubling of the width of the expectation value of the envelope is $t_N \simeq 2/n_0|C|\Delta p$. Alternatively, inserting Δp from (6), we find $t_N \simeq t_x$. The spreading of $\langle N(x,t) \rangle$ thus stems exlusively from the quantum-mechanical uncertainty regarding the position of the crest of the soliton—not from a degradation of the soliton.

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