Free energy of a noncritical superstring

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The free energy is derived in a modulus-invariant form for a noncritical superstring at a nonzero temperature. The Hagedorn temperature is determined.

Superstring theory at nonzero temperatures has attracted considerable interest (Refs 1–6; see Ref. 1 for a review of strings at nonzero temperatures). It appears that superstrings at nonzero temperatures should find applications in describing the early universe.

The basic thrust of Refs. 1–6 was to study the free energy, the existence of a Hagedorn temperature, and the interpretation of this temperature. As Rohm,[2] Gui-

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gain,[4] Likhttsier and Odintsov,[5] and Alvarez and Osorio[6] have shown, the superstring free energy can be written in modulus-invariant form. As we know, modulus invariance [the $SL(2,Z)$ invariance] is exceedingly important in string theory, since in its absence the systematic nature of string theory (anomalies) and gauge invariance would be disrupted. An $SL(2,Z)$ invariance also arises in field theory.[7]

Chamseddine[8] has recently proposed an interesting model of a noncritical boson string, which is based on an action of a two-dimensional quantum gravity of the form[8,9]

$$\int d^2x\sqrt{g}\Phi(R+\Lambda)+S_m, \tag{1}$$

where Φ is a scalar, R and Λ are the two-dimensional curvature and the two-dimensional cosmological constant, and S_m is the action of a σ-model type for d two-dimensional scalars. The free energy and the Hagedorn temperature for the model of Ref. 8 were found in Ref. 11 (see also Ref. 12) for nonzero temperatures for an arbitrary kind.

In this letter we derive the single-loop free energy for a model of a noncritical superstring[10] which is a supersymmetric generalization of the theory of Ref. 8.

The action has the following form in the superfield formulation:[10]

$$S=\frac{i}{2\pi}\int d^2zE\Phi(R_{+-}+K)+\frac{1}{4\pi}\int d^2zED_-X^iD_+X^i, \tag{2}$$

where $d^2zE=d^2xd\theta d\bar{\theta}\mathrm{sdet}E_M^A$ is the supervolume element, Φ is a dilation superfield, K is a constant, R_{+-} is the supercurvature, and $i=1,...,d$.

The introduction of a scalar superfield Φ in (2) restricts the supercurvature to a finite value. The partition function was calculated in all kinds in Ref. 10. It was shown that restrictions which lead to a critical dimension of 10 in standard superstring models do not arise in a theory with action (2). A theory with action (2) is therefore a noncritical-superstring theory which is systematic for arbitrary d.

Let us calculate the free energy for the theory in (2) at a nonzero temperature. We use the component formulation of the theory of (2) given in Ref. 10, its particle spectrum,[10] and the following expression for the free energy (Ref. 4; see also Ref. 5 and Rohm's paper[2]) in d dimensions:

$$F(\beta)=V^{d-1}\int_{-1/2}^{1/2}d\tau_1\int_{(1-\tau_1^2)^{1/2}}^{\infty}\frac{d\tau_2}{\tau_2}(4\pi\tau_2)^{-d/2}\{E_p(P_B-P_F)$$

$$+E_m(P_B+P_F)+O_p(P_B+P_F)(-\frac{1}{\tau}+O_m(P_m+P_F)[-\frac{1}{\tau+1}]\}. \tag{3}$$

Here V^{d-1} is the $(d-1)$-dimensional volume, β is the reciprocal temperature,

$$E_p = \sum_{n,m=-\infty}^{\infty} \exp(-\pi\beta^2|2n\tau + 2m|^2/\tau_2),$$

$$E_m = \sum_{n,m=-\infty}^{\infty} \exp\left(\frac{-\pi\beta^2|2n\tau+2m+1|^2}{\tau_2}\right),$$

$$O_p = \sum_{n,m=-\infty}^{\infty} \exp(-\pi\beta^2|(2n+1)\tau + 2m|^2/\tau_2),$$

$$O_m = \sum_{n,m=-\infty}^{\infty} \exp(-\pi\beta^2|(2n+1)\tau + 2m + 1|^2/\tau_2),$$

$P_B = \mathrm{tr}\{e^{-M^2\tau_2}[|1 + (-1)^F|/2]\}$, and $P_F = \mathrm{tr}\{e^{-M^2\tau_2}[|1 - (-1)^F|/2]\}$ are boson and fermion projection operators, respectively, and M^2 is a mass operator. Expression (3) leads to a modulus-invariant expression for the free energy.

After some straightforward, fairly standard calculations, we find

$$F(\beta) = \mathrm{const} \int_{-1/2}^{1/2} d\tau_1 \int_{(1-\tau_1^2)^{1/2}}^{\infty} \frac{d\tau_2}{\tau_2^2}(4\pi\tau_2)^{-d/2}|\theta_1'(0|\tau)|^{-d}$$

$$\times\{|\theta_2|^d E_m + |\theta_4|^d O_p + |\theta_3|^d O_m\}. \qquad (4)$$

Here the $\theta_i(0|\tau)$ are the Jacobi θ_i functions. Expression (4) for the free energy is modulus-invariant. In the $d = 8$ case it leads to the standard expression for a critical superstring.[2-5] It is a straightforward matter to verify that the integrand in (4) leads to the Hagedorn temperature

$$\beta_c = \pi\sqrt{d\alpha'}, \qquad (5)$$

where the α' dependence has been restored.

Expression (4) takes a particularly simple form in the $d = 0$ case:

$$F(\beta) = \mathrm{const} \int_{-1/2}^{1/2} d\tau_1 \int_{(1-\tau_1^2)^{1/2}}^{\infty} \frac{d\tau_2}{\tau_2^2}(E_m + O_p + O_m). \qquad (6)$$

It follows from (5) and (6) that in the $d = 0$ case we have $\beta_c = 0$ [in this case, the integral in (6) can be evaluated explicitly; cf. Refs. 11 and 13]. Consequently, with $d = 0$ (a two-dimensional supergravity) the Hagedorn temperature does not arise in the theory. Precisely the same result was found previously in a two-dimensional gravity.[11,13]

We have thus derived a modulus-invariant, single-loop free energy and the Hagedorn temperature for a noncritical superstring. It would be interesting to examine possible cosmological applications. Unfortunately, there is the obstacle that an analog of Einstein's equations has yet to be constructed for superstrings.

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