Neoclassical diffusion in a turbulent plasma

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A new diffusion mechanism is analyzed. It arises when electrostatic fluctuations and a toroidal drift of particles are taken into account simultaneously. The diffusion results from transitions of particles from trapped to untrapped trajectories. The diffusion coefficient is slightly higher than that in the neoclassical plateau regime, regardless of the collision rate.

A drift motion of particles in a fluctuating electromagnetic field appears to be responsible for the anomalous nature of the transport processes in tokamak plasmas. The plasma diffusion coefficients and the electron thermal conductivity are one or two orders of magnitude higher than their neoclassical values, while the ion thermal conductivity is larger by a factor of only a few units than its neoclassical value.[1] It is thus natural to suggest that neoclassical effects are not completely suppressed by the plasma turbulence and that these effects substantially influence the anomalous transport mechanism.

In the present letter we examine the diffusion of particles in a turbulent plasma, taking the toroidal drift into account. The toroidal drift acts primarily on the ions, for which the neoclassical transport is fairly rapid. We accordingly restrict the analysis to the ion component of the plasma. To analyze this component, it is sufficient to consider only the electrostatic component of the fluctuations; perturbations of the magnetic field can be ignored.

To describe the motion of the particles, we use the drift Hamiltonian

$$H(\vec{r}_\perp,t) = c\phi(\vec{r}_\perp,t)/B - u_d[\vec{e}_d(t),\vec{r}_\perp], \tag{1}$$

where $\phi(\vec{r}_\perp,t)$ is the electrostatic potential, B is the magnetic field $u_d = v_{ti}^2/\omega_B R$ is the toroidal drift velocity, and the unit vector $\vec{e}_d(t)$ changes direction over a time scale $\tau_t = 1/\omega_t = qR/v_{ti}$. This time scale is set by the motion of the particles along the tokamak magnetic field under the condition that the influence of electromagnetic fluctuations can be ignored for the longitudinal motion.[2]

Let us examine the nature of the motion of the particles in Hamiltonian (1). For the typical parameters of a tokamak plasma, the size of the fluctuations in the electrostatic potential in the main gradient region of the plasma column is[3] $e\phi/T \simeq 1/k_\perp a$, in order of magnitude, where $k = 1/\lambda$ is a transverse length scale of the fluctuations, and a is the minor radius of the torus. The velocity of the electrostatic particle drift in the fluctuations, $\tilde{v} \simeq ck\phi/B$, turns out to be higher than the neoclassical drift velocity:

$$\frac{\tilde{v}}{u_d} = \left(\frac{ck_\perp \phi}{B}\right) \left(\frac{v_{ti}^2}{\omega_B R}\right)^{-1} = \frac{R}{a} \gg 1. \tag{2}$$

If the time dependence of $\phi(\vec{r}_\perp,t)$ and $\vec{e}(t)$ is ignored, the particles move along equipotentials of the Hamiltonian H. Under the condition $\tilde{v} \gg u_d$, most of these equipotentials are closed and localized near extrema of ϕ. A small fraction of the trajectories, however, is unbounded along the toroidal drift direction and gives rise to a hydrodynamic percolation of particles at an average velocity u_d (Ref. 4).

To estimate the parameters of the percolation trajectories, we adopt the assumption that the fluctuating electrostatic potential is a function of a general type with a single length scale, λ. The contour curves of a potential of this sort are fractal curves, whose length L is a power-law function of the transverse dimension of a convection cell, Δ, under the condition $L \gg \lambda$ (Ref. 5)

$$L = \lambda(\Delta/\lambda)^d. \tag{3}$$

Here $d = 1 + 1/\nu$ is the fractal dimension, and $\nu = 4/3$ is the critical exponent for two-dimensional percolation.[6] Long trajectories, with $L \gg \lambda$, arise only for the contour lines, for which the value of the Hamiltonian H is considerably lower than the average value: $h \ll 1$, where $h = H/\langle H^2 \rangle^{1/2}$ is the relative height of the contour line. The maximum dimension of the convection cell formed by a contour line of this sort is

$$\Delta = \lambda h^{-\nu}. \tag{4}$$

The characteristic parameters of the percolation trajectories can be found from the condition that the average velocity of the medium, $v = |\langle \nabla H \rangle| = u_d$, is established on these contour lines:

$$f\bar{v} = u_d, \tag{5}$$

where $f \simeq h^\nu$ is the fraction of trajectories which have a size Δ, and $\bar{v} \sim \tilde{v}\Delta/L$ is the average directed velocity along the lines of a given size. From this expression we find estimates of the percolation level,[7]

$$h_d \sim (\tilde{v}/u_d)^{-1/(1+\nu)},$$

and of the average directed velocity,

$$\bar{v} \cong u_d(\tilde{v}/u_d)^{\nu/(\nu+1)}.$$

In systems which have trajectories of two types—passing trajectories and trajectories which are localized near extrema of H—the diffusion process is of the following nature. The particles on the passing trajectories are displaced a distance $\Delta \simeq \bar{v}\tau$ over the lifetime of these orbits. Over the time τ, a given passing trajectory is destroyed, and the particles moving along it switch to localized orbits. After a time $t \gg \tau$, these particles go back to passing trajectories, but now these trajectories are directed at random with respect to the original motion. This diffusion of the particles is illustrated by the trajectory in Fig. 1. The coefficient of the diffusion caused by this process can be estimated from $D \simeq f\tilde{v}^2\tau = u_d\tilde{v}\tau$.

For sufficiently narrow spectra of the electrostatic potential, $\Delta\omega \leqslant (a/R)\omega_* \simeq k\rho_i v_{ti}/R \simeq \omega_t$, (this is not an overly restrictive limitation for the fluctuations in a tokamak), the orbit lifetime τ is determined by the bounce frequency ω_t. The fluctuation diffusion coefficient of the ions can then be estimated from the following expression, where the toroidal drift is taken into account:

$$D \simeq \frac{u_d^2}{\omega_t}\left(\frac{\tilde{v}}{u_d}\right)^{4/7}. \qquad (6)$$

The first factor in this estimate corresponds to the neoclassical diffusion in the plateau regime; the second factor represents a very slight increase above the neoclassical value [with allowance for estimate (2) on the order of $(R/a)^{4/7}$].

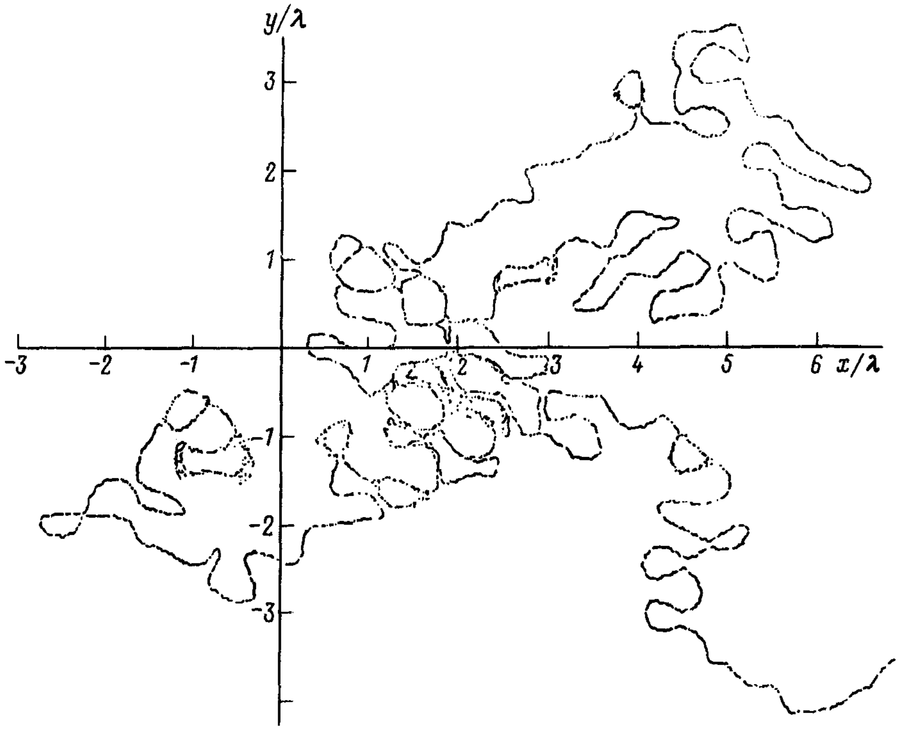

FIG. 1. Typical trajectory of a percolating particle under the conditions $\lambda\omega_t/u_d = 1$ and $\tilde{v}/u_d = 10$. Regions of the trajectory with fractal properties can be seen clearly. A capture to short orbits near extrema of the Hamiltonian (heavy lines) can also be seen clearly.

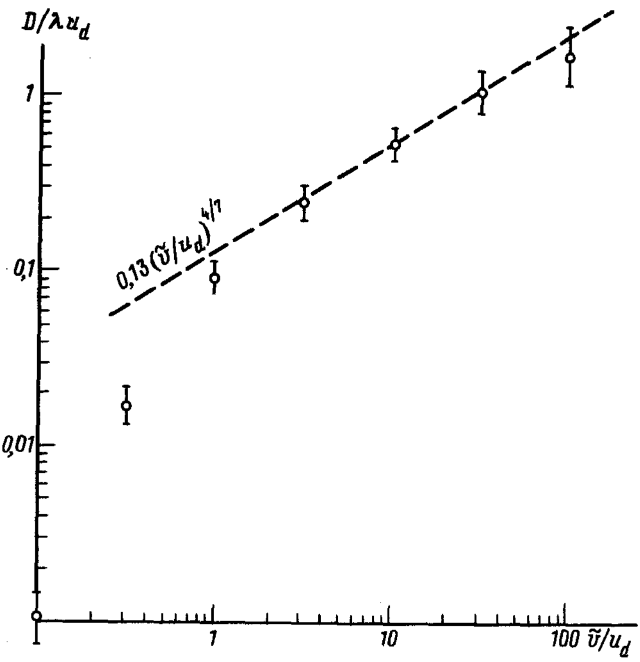

FIG. 2. The diffusion coefficient versus the relative fluctuation amplitude under the conditions $\lambda\omega_t/u_d = 1$ and $\Delta\omega = 0$.

Strictly speaking, the diffusion coefficient in (6) is valid only in the case $u_d/\lambda\omega_t = q\rho_i/\lambda \simeq 1$, in which there exists one length scale. When there is a substantial difference between $u_d\omega_t$ and λ, the diffusion process is greatly complicated by the presence of two length scales. To determine the diffusion coefficient, it will be necessary to carry out further studies. The condition $q\rho_i \simeq \lambda$, a not too stringent condition holds, however, for fluctuations in a tokamak.

For a quantitative determination of the diffusion coefficient, we carried out a numerical study of the motion of particles in Hamiltonian (1) under the conditions $u_d/\omega_t = q\rho_i \simeq \lambda$. As can be seen in Fig. 2, the functional dependence $D \simeq (\tilde{v}/u_d)^{4/7}$ is supported well by the numerical calculations. The sharp decrease in the diffusion coefficient at $\tilde{v} < u_d$ corresponds to the disappearance of the transitions (mentioned above) between two types of orbits.

In summary, when toroidal drift and electrostatic fluctuations are taken into account simultaneously, a new diffusion mechanism arises. This mechanism is a consequence of transitions from localized to moving trajectories. It arises only at relatively high fluctuation amplitudes ($\tilde{v} > u_d$ or $e\phi/T > \lambda/R$). This mechanism operates even for steady-state perturbations ($\Delta\omega = 0$). It gives rise to a diffusion with a coefficient greater than the coefficient corresponding to the neoclassical plateau, regardless of the collision rate. The transport coefficient in (6) can thus explain both the very small magnitude of the anomaly in the ion thermal conductivity at moderate collision rates and the experimentally observed increase in the anomalous transport in the banana regime [see Eq. (1)].

For fluctuations with a finite spectral width, $\Delta\omega \gg \omega_t$, the diffusion coefficient becomes the expression derived in Ref. 7:

$$D \cong \frac{u_d^2}{\omega_t}\left(\frac{\tilde{v}}{u_d}\right)^{7/10}\left(\frac{\Delta\omega}{\omega_t}\right)^{3/10}\left(\frac{\lambda\omega_t}{u_d}\right)^{3/10}.$$

As would be expected, the joining is smooth only under the condition $u_d/\lambda\omega_t = q\rho_i/\lambda \simeq 1$. In this case the problem has only a single length scale.

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