Factorization of the K^0–$\bar{K}^0$-mixing matrix element in left–right-symmetry theories

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A sum-rule method is used to evaluate the accuracy of the vacuum dominance hypothesis for the matrix element representing K^0–$\bar{K}^0$ mixing in left–right-symmetry theories.

The standard model[1] has weathered many precise experimental tests.[2] The precision of these experiments has been such that the radiation corrections in terms of the gauge coupling constants must be taken into account systematically in the corresponding theoretical calculations.[3] It is believed that the structure of these corrections can serve as a subtle test of the overall standard-model scheme. It is also believed that a quantitative comparison with experimental data can be useful for determining parameters of the standard model which are still unknown, such as the mass of the t quark. In addition to successfully confirming various predictions of the standard model, experimentalists are conducting active searches for manifestations of physical effects stemming from deviations from the standard model.[4] One direction of this search is to attempt to observe additional new gauge bosons.[5] Expansions of the standard model containing new gauge bosons arise in various models, e.g., in the grand unification theories[6] and in models associated with a low-energy limit of superstring theories.[7] An extremely simple generalization of the standard model is the left–right-symmetry model based on the $SU_L(2)\times SU_R(2)$ group.[8] To find limitations on the masses of right-handed gauge bosons W_R, we could make use of their contribution to the difference between the masses of neutral kaons, which is known experimentally. To calculate this contribution theoretically, it is necessary to evaluate the matrix element of a local four-quark operator which arises when the short-range contribution is distinguished from the known box diagram, between kaon and antikaon states. Without the corrections for the strong interactions, this matrix element is $\langle K^0|\hat{O}_{LR}|\bar{K}^0\rangle$, where $\hat{O}_{LR} = \bar{s}_R d_L \bar{s}_L d_R$. Numerical estimates have been found by the method of three-point sum rules by Colangero and Nardulli,[9] who observed a pronounced (fivefold!) violation of the vacuum dominance hypothesis[10] for this matrix element. The result looks unnatural. Our objection is that the vacuum dominance hypothesis for the matrix element is based on the use of Wick's theorem in the approximation of free quark fields. The same approximation, i.e., free quark fields and Wick's theorem, is used as a leading approximation within the framework of the sum-rule method.[11,12] A pronounced violation of the vacuum dominance hypothesis in this case would signify large corrections for higher-degree terms in the asymptotic expansion or for large perturbative contributions of strong interactions to the coefficient functions of local operators within the framework of the sum-rule method. In turn, this circumstance casts doubt on the validity of the sum-rule method itself for quantitative determinations of the matrix element. In the present letter we show that the vacuum dominance hypothesis works completely satisfactorily for the matrix element of the operator $\hat{O}_{LR}$, i.e., $\langle K^0|\hat{O}_{LR}|\bar{K}^0\rangle$. Our result differs substantially from that found in Ref. 9. We discuss the reasons for these discrepancies briefly in the conclusion to this letter.

In the vacuum dominance approximation, we find the following expression for the matrix element:

$$< K^0|\hat{O}_{LR}|\bar{K}^0 >= -\frac{1}{2}(g_K^2 + \frac{1}{2N_c} f_K^2 m_K^2) = -\frac{1}{2}(\frac{f_K m_K^2}{m_s + m_d})^2 (1 + \frac{1}{2N_c}\frac{(m_s + m_d)^2}{m_K^2}),$$

$$g_K = \frac{f_K m_K^2}{m_s + m_d} = -\frac{< \bar{s}s > + < \bar{d}d >}{f_K}. \qquad (1)$$

In the chiral limit, i.e., when terms proportional to the quark masses m_q are ignored, the result is

$$< K^0|\hat{O}_{LR}|\bar{K}^0 >^{\text{ch lim}} = -\frac{1}{2} g_K^2 = -\frac{1}{2}\left(\frac{< \bar{s}s > + < \bar{d}d >}{f_K}\right)^2 .$$

The part which is suppressed in the chiral limit amounts to only 2% of the overall result. The reason for this small fraction is not simply the suppression due to the small masses of the light d and s quarks but also a further suppression of this contribution by virtue of the number of colors. It can be seen from estimate (1) that calculations in the chiral limit of strong interactions are sufficient for testing the vacuum dominance hypothesis.

In order to derive a reliable result by the sum-rule method, we need to accurately distinguish the contribution from the matrix element of interest here to the Green's function used in the calculation of the operator expansion. Let us consider the Green's function

$$T^{\mu\nu} = \int < 0|Tj^\mu(x)\hat{O}_{LR}(0)j^\nu(y)|0 > e^{ipx-ip'y}dxdy = p^\mu p'^\nu T(p^2, p'^2, pp') + ..., \tag{2}$$

where $j^\mu = \bar{d}\gamma^\mu\gamma_5 s$. In the leading approximation, the asymptotic expansion of the amplitude $T(p^2,p'^2,pp')$ in the Euclidean region of the range of the variables p,p'^2, as p^2, $p'^2 \to \infty$, is

$$T(p^2, p'^2, pp') = \frac{< O_4 >}{p^2 p'^2} + ...,$$

$$< O_4 > = 2 < (\bar{d}_R d_L + \bar{s}_R s_L)(\bar{d}_L d_R + \bar{s}_L s_R) > + < \bar{s}_R d_L \bar{d}_L s_R > + < \bar{s}_L d_R \bar{d}_R s_L > . \tag{3}$$

We have omitted from (3) a term which leads to a contribution to the matrix element which has a chiral suppression. Assuming that a vacuum factorization is valid for evaluating the vacuum expectation values of four-quark operators, we find

$$< O_4 >^{fact} = \frac{1}{2}(< \bar{s}s > + < \bar{d}d >)^2 \tag{4}$$

or

$$T(p^2, p'^2, pp') = 2\Pi(p)\Pi(p'), \tag{5}$$

where

$$\int < 0|Tj^\mu(x)\bar{s}_R d_L(0)|0 > e^{ipx}dx = p^\mu \Pi(p), \quad \Pi(p) = \frac{1}{2}\frac{< \bar{s}s > + < \bar{d}d >}{p^2}. \tag{6}$$

The leading contribution to the asymptotic behavior of Green's function (2) can thus be factorized completely.

We wish to stress that the result in (3), with approximation (4) and representation (5), is completely natural, since it was derived on the sole basis of Wick's theorem for free quark fields. If we restrict the discussion to the order of the asymptotic expansion which we have obtained for Green's function (2), and if we determine the matrix element through the use of, say, local duality,[13] we find that vacuum dominance holds exactly for the matrix element. Expression (6) for the correlation function $\Pi(p)$ gives the exact result in the chiral limit if we ignore the contributions of many-particle states, e.g., the three-kaon contribution.

We thus see that corrections to the vacuum dominance approximation for the matrix element arise from several sources. There are corrections for the operators of higher dimensionality, the purely perturbation-theory corrections to the coefficient function of the leading four-quark operator, and the corrections for so-called bilocal contributions. In principle, all these corrections can be calculated, but there are still uncertainties stemming from the approximate estimate of the vacuum expectation values of the local operators. For example, the accuracy of approximation (4) for the four-quark operators which determine the leading contribution to the matrix element for the LL operator $\bar{s}_L \gamma^\mu d_L \bar{s}_L \gamma_\mu d_L$, was evaluated in Ref. 14 and was found to be about 20%.

The leading power-law correction to expression (3) is given by the vacuum expectation values of the local operators of dimensionality 8, e.g., the operator $\bar{s}_R \sigma^{\mu\nu} d_L \bar{s}_L G_{\mu\nu} d_R$, of the stress tensor of the gluon field, $G_{\mu\nu}$. The vacuum expectation values for all such operators turn out to be zero in the vacuum dominance approximation. We will ignore all corrections of order α_s to the coefficient functions of the local operators here, with the intention of taking them up in a more comprehensive paper. The coefficient function of a bilocal operator is determined by an asymptotic expansion of the product $\int Tj^\mu(x) j^\nu(0) e^{ipx} dx$. It turns out to be suppressed by a factor of α_s, and we will ignore it in our approximation.

Going through the standard procedure for analyzing the sum rules,[15] we thus find that the vacuum dominance hypothesis works in the chiral limit, since there is nothing to cause a pronounced deviation from this hypothesis. Incorporating small corrections for the nonzero masses of the d and s quarks obviously could not cause any substantial change in the calculated results. A more detailed analysis shows that the leading contribution in terms of the parameters representing the breaking of chiral symmetry (the masses of the quarks) can also be factorized completely. A corresponding analysis can be carried out for the operator $\hat{O}'_{LR} = \bar{s}_R \gamma^\mu d_R \bar{s}_L \gamma_\mu d_L$, which mixes with the operator $\hat{O}_{LR}$ when the single-loop gluon corrections are taken into consideration. The leading asymptotic expression for the corresponding scalar amplitude of the Green's function, which contains a contribution to the matrix element of the operator $\hat{O}'_{LR}$, without a chiral suppression, is

$$T^{\mu\nu}(p^2, p'^2, pp') = \int <0|Tj^\mu(x)\hat{O}'_{LR}(0)j^\nu(y)|0> e^{ipx-ip'y}dxdy$$

$$p^\mu p'^\nu \frac{1}{N_c}\frac{(<\bar{s}s> + <\bar{d}d>)^2}{p^2 p'^2} + \dots \ .$$

It makes the vacuum dominance hypothesis valid. We wish to stress that calculations in the chiral limit for this operator are less reliable, since the leading contribution is suppressed in the calculation in $1/N_c$:

$$<K^0|\hat{O}'_{LR}|\bar{K}^0> = -\frac{1}{2}\left(\frac{2}{N_c}g_K^2 + f_K^2 m_K^2\right).$$

Let us take a brief look at the distinctions between these calculations and those of Ref. 9. Colangero and Nardulli[9] did not draw distinctions between contributions with different degrees of chiral suppression. The choice of a kaon interpolation operator in the form $j = \bar{d}\gamma_5 s$ for the operator $\hat{O}_{LR} = \bar{s}_R d_L \bar{s}_L d_R$ leads to difficulties in distinguishing the leading factorizable contribution. Specifically, it becomes necessary to devise an accurate procedure for subtracting, from the total three-point Green's function, a product $\Pi_s(p)\Pi_s(p')$ of two nontrivial (in contrast with our approach) correlation functions of the type

$$\Pi_s(p) = \int < 0|Tj(x)\bar{s}_R d_L(0)|0 > e^{ipx} dx$$

with the asymptotic behavior

$$\Pi_s(p) = \frac{3}{16\pi^2} p^2 \ln(p^2/\mu^2) + \frac{< \alpha_s G^2 >}{16\pi p^2} - \frac{44\pi\alpha_s < \bar{q}q >^2}{27p^4} + \dots .$$

The product $\Pi_s(p)\Pi_s(p')$ is a completely factorizable contribution. It evidently leads to the value $B = 1$, which is the value dictated by the vacuum dominance hypothesis. This result cannot, however, be derived in a simple way through a direct numerical analysis of the sum rules. One possibility for testing the accuracy of the method used in Ref. 9 would be to check whether the value $B = 1$ is reproduced for the product $\Pi_s(p)\Pi_s(p')$.

In a more detailed paper we hope to discuss in detail the limitations on the mass of the right-handed gauge bosons which follow from our result.

In conclusion we wish to repeat that a successful choice of interpolating currents for kaons has made it possible, in our approach, to distinguish a factorizable part of the asymptotic Green's function and to show that the corrections to this function which are not factorizable cannot be large.

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