Anomalous sign of the polaron effect for excitons interacting with an incompressible fluid

V. M. Apal'kov and É. I. Rashba

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, 117940, Moscow

(Submitted 14 November 1991)

Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 1, 38–43 (10 January 1992)

A dispersion relation is derived for a magnetoexciton interacting with a 2D incompressible fluid. In an asymmetric system, the shift of the exciton level caused by this interaction has the sign opposite that of the ordinary polaron shift. When the asymmetry parameter reaches a critical value, the absolute minimum of the exciton spectrum is displaced to a circle. This effect results in an abrupt change in the properties of the emission spectrum. A model for the new ground state is proposed.

It has been shown previously,[1] in two examples, that some symmetry limitations prevail in the magnetospectroscopy of two-dimensional (2D) electron systems. When these limitations are satisfied, the electron-electron interaction drops out of the frequency region of interband absorption and luminescence, and the transition frequencies have the same values as in an "empty" crystal. These comments apply to both exciton transitions[2] and the radiative capture of electrons from a 2D channel by a neutral impurity in a confinement plane. For a perfect crystal, the conditions for the occurrence of a hidden symmetry responsible for the universal nature of optical spectra are as follows: (1) There is charge symmetry, by which we mean that all the electron and hole interaction potentials are equal in magnitude ($V_{ee} = V_{hh} = -V_{eh}$). (2) All the charge carriers are in the lower level of electric and magnetic quantization. (3) Mixing of levels can be ignored. We wish to stress that the most interesting effects which occur in a 2D gas can be described within the framework of this model: the formation of an incompressible fluid,[3] charge density waves, etc. However, these effects are not manifested in optical spectra. The interband spectra can thus provide nontrivial information on the processes which occur in the electron system only under conditions such that this hidden symmetry is removed. A model of extrinsic recombination from a 2D system[4] was analyzed in Ref. 1. The nature of the singularity in the position of the emission band as a function of the filling factor was found (a downward cusp). It is associated with a cusp in the ground-state energy.[5] The size of the gaps in the spectrum of fluids was determined in Ref. 6 on the basis of the theory of Ref. 1; reasonable results were found. In the present letter we derive a theory for magnetoexcitons under conditions of a broken charge symmetry.

We introduce plasmon operators for the electrons:

$$a(\vec{q}) = (2\pi/A)^{1/2} \sum_p a^+_{p-q_y/2} a_{p+q_y/2} \exp(ipq_x), \tag{1}$$

where $a^+(\vec{q}) = a(-\vec{q})$, and we introduce corresponding operators $b(\vec{q})$ for holes. Here a_p are electron operators in the Landau gauge; $A_y = x; A_x = A_z = 0, \vec{q}$ is the momentum; the unit of length is the magnetic length $l(H)$; and A is the size of the normalization region. It is also convenient to define the operators

$$c(\vec{q}) = a(\vec{q}) - b(\vec{q}), \quad d(\vec{q}) = a(\vec{q}) + b((\vec{q}), \tag{2}$$

where the operator $c(\vec{q})$ represents the electric charge density. The Hamiltonian of the electron-hole system is then given by (energies are expressed in units of $e^2/\epsilon l$)

$$H = H_1 + H_2, \tag{3}$$

$$H_1 = \frac{1}{4\pi}\sum_{\vec{q}}\{\tilde{V}_s(q)c(\vec{q})c(-\vec{q}) + \tilde{V}_a(q)d(\vec{q})d(-\vec{q}) + \tilde{v}(q)(c(\vec{q})d(-\vec{q}) + d(\vec{q})c(-\vec{q}))\}, \tag{4}$$

$$H_2 = -\frac{1}{2}\{\tilde{V}_e\hat{N}_e + \tilde{V}_h\hat{N}_h\} \tag{5}$$

$$\tilde{V}_e = \int d\vec{q}\tilde{V}_{ee}(q)/(2\pi)^2, \quad \tilde{V}_h = \int d\vec{q}\tilde{V}_{hh}(q)/(2\pi)^2. \tag{6}$$

Here the operators $\hat{N}_e$ and $\hat{N}_h$ represent the numbers of electrons and holes. The potentials $V_s(q)$, $V_a(q)$, and $v(q)$ are given in the momentum representation by

$$V_s = (V_{ee} + V_{hh} - 2V_{eh})/4, \quad V_a = (V_{ee} + V_{hh} + 2V_{eh})/4, \quad v = (V_{ee} - V_{hh})/4. \tag{7}$$

The tilde ($\sim$) on the potentials in (5) means that they are multiplied by a factor of $\exp(-q^2/2)$; for a Coulomb potential we would have $V(q) = 2\pi/q$. For a charge-symmetric system we would have $V_s(q) = V(q)$ and $V_a = v = 0$, and the operators $d(\vec{q})$ would drop out of the Hamiltonian.

We not introduce an operator which annihilates an exciton with momentum k:

$$\mathcal{A}(\vec{k}) = (2\pi/A)^{1/2}\sum_p a_{p+k_y/2}\beta_{-p+k_y/2}e^{ipk_x}, \tag{8}$$

where the operator β_p annihilates a hole. The following commutation relations hold:

$$[c(\vec{q}), \mathcal{A}(\vec{k})] = 2i(2\pi/A)^{1/2}\sin((\vec{k}\times\vec{q})/2)\,\mathcal{A}(\vec{k}+\vec{q}), \tag{9}$$

$$[d(\vec{q}), \mathcal{A}(\vec{k})] = -2(2\pi/A)^{1/2}\cos((\vec{k}\times\vec{q})/2)\,\mathcal{A}(\vec{k}+\vec{q}), \tag{10}$$

where $(\vec{k}\times\vec{q}) = k_x q_y - k_y q_x$. It follows from (9) and (10) that there is an important distinction between the exciton spectra in symmetric and asymmetric systems. The distinction stems from the circumstance that $\mathscr{A}(\vec{k}=0)$ commutes with $c(\vec{q})$ but not with $d(\vec{q})$. For the former, with ψ being an eigenfunction of the system with the energy ϵ, the quantity $\Psi = \mathscr{A}^+(\vec{k}=0)\psi$ is an eigenfunction of the system which differs from the original one in that there is an exciton with a momentum $\vec{k}=0$ and an

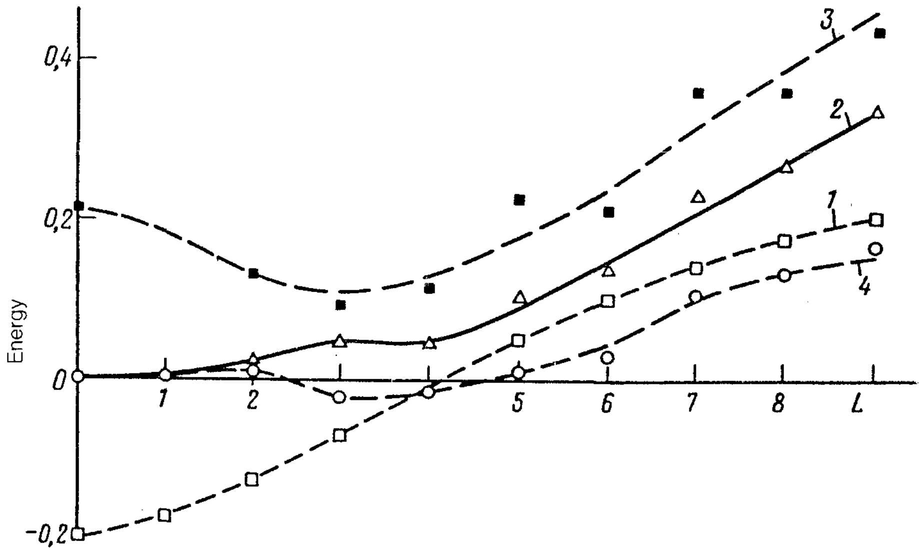

FIG. 1. Energy versus the angular momentum L. For all points 1–3, the value of h is 0.5, 1,□—Energy of a bare exciton; 2,*—energy of a dressed exciton; 3,■—energy of magnetorotons. Points 4 (O) show the energy of a dressed exciton for the value $h = 1.0$. The origin of the energy scale is placed at the energy of a dressed exciton with $k = 0$. The curves are simply drawn through the points to aid the eye.

energy $E = \epsilon + E_{ex}(\vec{k} = 0)$ (Ref. 2). In particular, if the exciton arises against the background of an incompressible fluid, then its interaction with the fluid can be treated as a polaron effect, resulting from a dressing by magnetorotons.[7] In this case, the formula found for the energy here means that the polaron shift is zero at $\vec{k} = 0$ in symmetric systems. This fact was discussed in Ref. 2; the assertion applies to both charged and neutral systems.

We have studied the energy spectrum of an exciton against the background of an incompressible fluid on the basis of Hamiltonian (3)–(5) in spherical geometry.[8] Most of the calculations were carried out for a system consisting of $N = 4$ electrons plus 1 exciton. We assumed a filling factor $\nu = 1/3$. The radius of the sphere is $R = [(N-1)/2\nu]^{1/2} = 3/2^{1/2} \approx 2.1$. The angular momentum L and the momentum k are related by $L = Rk$. Figure 1 shows the energy spectrum of the bare exciton, $E(L)$ (i.e., the spectrum of an exciton in an empty crystal). Also shown here are the renormalized exciton spectrum $E^*(L)$ and the roton spectrum ω_{mr}. The distance between the confinement planes for electrons and holes is $h = 0.5$.

The arrangement of levels in the $\vec{k} = 0$ case is unusual, in that the energy of a bare exciton lies below that of a dressed exciton. This situation means that the polaron shift of the level is positive. The reason for this anomaly is a specific feature of this problem, which distinguishes it from the ordinary polaron problem: The electron belonging to the exciton and the electrons of the fluid form a common Fermi sea. The limitations imposed by the Pauli principle raise the level of the exciton, since the presence of the fluid excludes a large fraction of the phase volume. This contribution competes with the ordinary polaron effect, which tends to lower the level. In the symmetric system, the two cancel out exactly, while in an asymmetric system the sign of the level shift is determined by the Pauli contribution.

In the case $h=0$ we have $E^*(L=0) \approx E^*(L=1)$. On this basis, it can be suggested[2] that the expansion of $E^*(k)$ does not contain terms on the order of k^2. At values $h \neq 0$, terms quadratic in k are unconditionally present in $E^*(k)$.

The $E^*(L)$ dependence changes qualitatively at $L \approx 3$, i.e., as E^* approaches the energy of a roton, $E^*(L) \approx \omega_{mr}(L)$. The spectrum does not end at the threshold, but the nature of the states changes: At $L \geqslant 4$, they may be thought of as bound states of a roton with a slow exciton.[9] Evidence in favor of this interpretation comes from the nearly parallel course of curves 2 and 3; the distance between them represents a binding energy. This interpretation can also be tested by expanding the exact wave function in the states to which the bare exciton contributes various angular momenta L'. The contributions of the states $L'=0$, 1, and 2 to the normalization integral is 0.68 for a total angular momentum $L=4$; it is 0.86 for $L=6$, and it is 0.82 for $L=8$. A point of importance is that all three states make comparable contributions. At $L=6$, for example, these contributions are 0.51, 0.23, and 0.12 for $L'=0$, 1, and 2, respectively. The $L'=0$ state makes the largest contribution. The picture is less convincing at $L=5$ and 7, at which the contribution of states with $L'=0$ is very small. This result is apparently an artifact resulting from the small number of particles ($N=4$); for the same

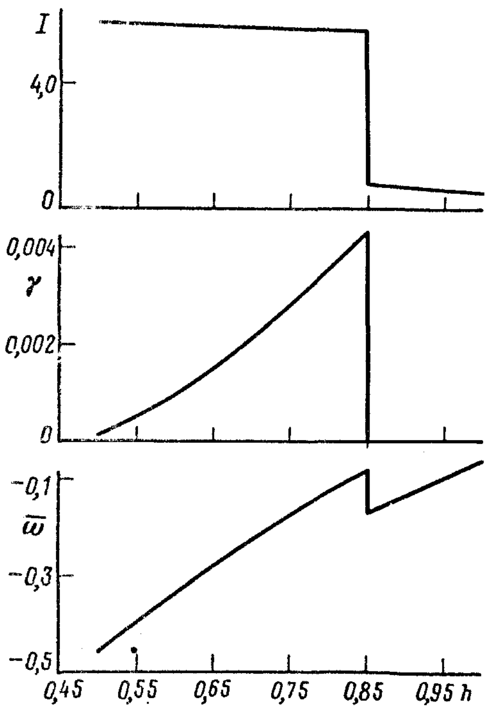

FIG. 2. Basic characteristics of the exciton band as a function of h. $\bar{\omega}$—Position of the band; γ—width of the band; I—intensity of the band.

reason, the frequencies of rotons with $L=5$ and 7 deviate from the general course of curve 3 (Fig. 1). We wish to stress that for $h=0$ there are never any terms with $L=0$ in the corresponding expansion, since an exciton with $\vec{k}=0$ does not interact with rotons.

An interesting aspect of curve 2 in Fig. 1 is the inflection point. At large values of h, it converts into a minimum, and at $h \gtrsim 0.85$ this minimum becomes the absolute minimum (cf. curve 4). The meaning here is that the minimum of the spectrum is displaced onto a circle with a radius $k \gtrsim 1.5$. Here we see an analogy with the onset of charge density waves in double quantum wells (see Refs. 10–12 and the references there). At $T=0$, this displacement of the minimum should be accompanied by an abrupt change in the emission spectrum. When the minimum is at $k=0$, the roton-free transition has the highest intensity. After the displacement of the minimum, this roton-free transition becomes forbidden, because of momentum conservation, and the one-roton band becomes predominant. As a result, the position of the band changes abruptly, and its intensity falls sharply. The results of these calculations are shown in Fig. 2.

The change which occurs in the distribution of the electron density screening the hole upon this shift of the minimum of the spectrum is interesting. The correspondence between plane geometry and spherical geometry is established by averaging this density over all values of M (the projection of the angular momentum) at a fixed L. After this averaging is made, the charge distribution around a hole on the sphere at the

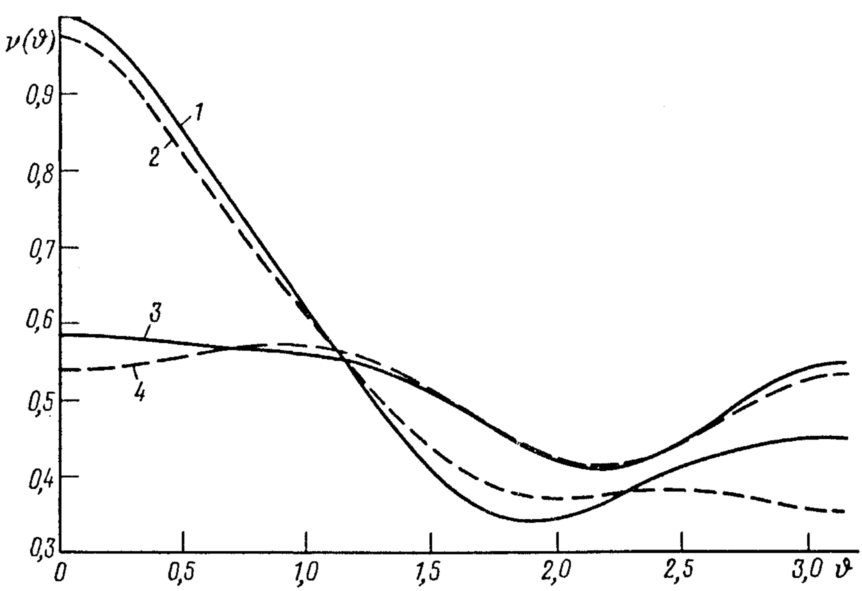

FIG. 3. Distribution of the electron density $\nu(\vartheta)$ in an exciton for various distances (h) from the hole to the electron confinement plane. The four curves correspond to the following values of h, of the angular momentum in the ground state (L), and of the net charge in the northern hemisphere (Q): 1—$h=0$, $L=0$, $Q \approx 1.06$; 2—$h=0.845$, $L=0$, $Q \approx 1.05$; 3—$h=0.85$, $L=3$, $Q \approx 0.67$; 4—$h=1.0$, $L=3$, $Q \approx 0.67$.

point $\vec{\omega}_h$, does not depend on $\vec{\omega}_h$. It is convenient to express the density in units of $n_0(S) = (2S+1)/4\pi S$, which is the Fermi limit on a sphere of radius $R = S^{1/2}$. When the hole is at the north pole $\vartheta_h = 0$; ϑ is the polar angle), the distribution of the electron density is given by

$$\nu(\vartheta) = \frac{4\pi}{2L+1} \sum_M \int |\Psi_{LM}(\vec{\omega}_1 ... \vec{\omega}_{N+1} | \vec{\omega}_h)|^2 \frac{1}{S} \sum_{j=1}^{N+1} \delta(\vec{\omega} - \vec{\omega}_j)\, d\vec{\omega}_1 ... d\vec{\omega}_{N+1} / n_0(S), \tag{11}$$

which has the meaning of a filling factor. Figure 3 shows the results calculated for the function $\nu(\vartheta)$ in the ground state of the system for four values of h. The density of the screening charge is found from $\nu(\vartheta)$ by subtracting $\nu = N/(2S+1)$. The shift of the minimum of the spectrum is accompanied by a dramatic change in $\nu(\vartheta)$. At $h = 0$ we have $L = 0$ in the ground state, as is true in general of all multiplicative states,[2] and $\nu(0) = 1$. With increasing h, the quantity $\nu(0)$ decreases slowly as long as the minimum remains at $L = 0$. Also varying slowly is the total screening charge Q, which we define somewhat arbitrarily as the integral of $n_0(S)(\nu(\vartheta) - \nu)$ over the northern hemisphere. Its value is approximately 1 as long as the minimum remains at $L = 0$; the value falls to $Q \approx 2/3$ when the minimum shifts to the point $L = 3$. The following model might be suggested for the new ground state of the exciton: a neutral formation consisting of a hole screened by three quasielectrons, two at a distance of approximately $l(H)$ from the hole, and a third at a greater distance. The latter distance cannot be found for the particular sphere size which we are using.

[1] V. M. Apal'kov and É. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 420 (1991) [JETP Lett. **53**, 442 (1991)].

[2] V. M. Apal'kov and É. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **54**, 160 (1991) [JETP Lett. **54**, 155 (1991)].

[3] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[4] I. Kukushkin, V. Timofeev, K. von Klitzing, and K. Ploog, *Advances in Solid State Physics* (Festkörperprobleme), Vol. 28, 1988, p. 21.

[5] B. I. Halperin, Helv. Phys. Acta b56, 75 (1982).

[6] Kh. Buman, V. Ioss, I. V. Kukushkin *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 426 (1991) [JETP Lett. **53**, 449 (1991)].

[7] S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).

[8] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).

[9] I. B. Levinson and É. I. Rashba, Usp. Fiz. Nauk **111**, 683 (1973) [Sov. Phys. Usp. **16**, 892 (1974)].

[10] W. A. Fertig, Phys. Rev. B **40**, 1087 (1989).

[11] D. Yoshioka and A. H. MacDonald, J. Phys. Soc. Jpn. **59**, 4211 (1990).

[12] X. M. Chen and J. J. Quinn, Phys. Rev. Lett. **67**, 895 (1991).

Translated by D. Parsons