Stimulation of superconductivity by a quasiparticle current in a multilayer superconducting tunnel structure

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A two-barrier tunnel structure of the type $S_h ISIS_h$, in which the superconductors $S_{(h)}$ differ substantially in (equilibrium) transition temperature ($T_{c0} \ll T_c^h$), is analyzed. The gap Δ in S is found as a function of the voltage across the structure, V. Several results are found: (1) At $eV < 2(\Delta_h + \Delta_0)$, the gap may grow to values on the order of the equilibrium value $\Delta_0 \sim T_{c0}$. (2) At $T > T_{c0}$, the solution for Δ is a double-valued function of V and T. (3) At $eV < 2(\Delta_h - \Delta_0)$, three phase transition may occur as T is lowered, at temperatures $T_{c1} > T_{c2} > T_{c3}$. Since the solution is multivalued, there are structural features on plots of the quasiparticle component and the Josephson component of the current versus V and on T.

In some recent experiments, Blamire *et al.*[1] passed a quasiparticle current through an $S_h ISIS_h$ tunnel structure ($S_{(h)}$ is a superconductor with an equilibrium transition temperature $T_{c0} \ll T_c^h$, and I is a layer of an insulator). They observed a substantial stimulation of superconductivity in the S layer at temperatures $T > T_{c0}$. The possibility of such a stimulation had been discussed previously at a qualitative level by Parmenter,[2] who pointed out that this stimulation would result from an extraction of quasiparticles from S which would occur at certain voltages. In the theoretical analysis which is the content of the present letter, it is shown that at low barrier transmission coefficients (as are realized experimentally), at which the proximity effect is inconsequential, the gap in S can grow to values on the order of the equilibrium value Δ_0, corresponding to $T = 0$, at voltages in the range $eV < 2(\Delta_h + \Delta_0)$ (Δ_h is the gap in superconductor S_h). It turns out that Δ is a double-valued function of V and T. It is shown that for each value $V < 2(\Delta_h + \Delta_0)/e$ there can be from one to three values of T_{cm} ($m = 1$–3). The two highest values determine the temperature interval ($T_{c1} \leqslant T \leqslant T_{c2}$) in which there is a solution that vanishes at the endpoints discontinuously (there is a first-order phase transition). The third value (if it exists) corresponds to a second-order phase transition. We will discuss some interesting aspects of the quasiparticle component and the Josephson component of the current which stem from this dependence of Δ on the voltage and the temperature.

In describing this structure, we work from a system of equations for semiclassical Green's functions,[5,6] as in Refs. 3 and 4. We join these equations at the barriers through the use of boundary conditions.[7] We assume the dirty limit: The mean free path satisfies $l \ll d, \xi$, where d is the thickness of layer S. We can find an equation for the matrix Green's function[6] $\langle \check{G}_p \rangle = \check{G}$ integrated over angles and for the boundary

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condition for it.[8] Working from those results, and assuming $d \ll \xi$, we easily derive an equation for $\check{G} \equiv \check{G}(t,t')$ (we are assuming that the electric potential of superconductor S is zero):

$$i\hbar \check{\tau}_z \partial \check{G}/\partial t + i\hbar \partial \check{G}/\partial t' \check{\tau}_z - \check{\Delta}(t)\check{G} + \check{G}\check{\Delta}(t') + \check{\Sigma}\check{G} - \check{G}\check{\Sigma} = 0, \tag{1}$$

$$\check{G}^2(t,t') \equiv \int dt_1 \check{G}(t,t_1)\check{G}(t_1,t') = \check{1}\,\delta(t-t'), \tag{2}$$

where $\check{\Sigma} = i\epsilon_1\check{G}_1 + i\epsilon_2\check{G}_2 + \check{\Sigma}_{ph}$, $\epsilon_j = \hbar D_j v_F/4d$, D_j is the transmission coefficient averaged over angles (with a certain weight[3,4,8]), $\check{G}_{1(2)}$ is the Green's function of superconductor S_h on the left (right) of S (we assume that this function is the equilibrium function; we can arrange this condition by means of a small value of D_j and a large thickness of the S_h layers), $\check{\Sigma}_{ph}$ is the eigenenergy matrix determined by the electron-phonon interaction in S, and $\check{\Delta}(t)$ is the order-parameter matrix.[6] The dimensions of the structure in a direction transverse with respect to the current are assumed to be small (in particular, small in comparison with the energy relaxation length), so there is no spatial nonuniformity. Knowing the Green's function $\check{G}$, and working from the boundary condition, we easily find an expression for the current through barrier j, with a resistance R_j ($j = 1,2$):

$$I_j(t) = (\pi/8eR_j)(-1)^j \mathrm{Tr}\hat{\tau}_z[\hat{G}_j^R\hat{G} + \hat{G}_j\hat{G}^A - \hat{G}^R\hat{G}_j - \hat{G}\hat{G}_j^A](t,t). \tag{3}$$

Before we take up the effect on Δ of a deviation from equilibrium, let us consider very briefly another factor: the proximity effect, which stimulates a superconductivity in S *under equilibrium conditions*. By virtue of this effect, a supercurrent[3,8,9] can flow through this structure at $T > T_{c0}$. This current is related to the difference between the phases of the order parameters across the structure, φ, at $V = 0$ by[9] (with $R_1 = R_2$)

$$eRI_s(\varphi) = \sin\varphi\, 2\pi T \sum_{n>0} (\epsilon_b f_h(\omega_n) + \tilde{\Delta}) f_s(\omega_n)/\varsigma_n. \tag{4}$$

Here $\tilde{\Delta}\cos(\varphi/2) = \Delta$, and $\tilde{\Delta}$ satisfies the equation

$$[\ln(T/T_{c0}) + 2\pi T \sum_{n\geq 0}(1/\omega_n - 1/\varsigma_n)]\tilde{\Delta} = \epsilon_b 2\pi T \sum_{n\geq 0} f_s(\omega_n)/\varsigma_n, \tag{5}$$

where $\zeta_n = \{[\omega_n + \epsilon_b g_h(\omega_n)]^2 + [\epsilon_b f_h(\omega_n) + \tilde{\Delta}]^2\cos^2(\varphi/2)\}^{1/2}$, $\omega_n = \pi T(2n + 1)$, $g_h(\omega)$ and $f_h(\omega)$ are the temperature Green's functions of superconductor S_h, and $\epsilon_b = \hbar D v_F/2d$. In principle, we could work from Eqs. (4) and (5) to find the $I_s(\varphi)$ dependence for arbitrary T and for an arbitrary relationship between ϵ_b and T_{c0} (Ref. 9). We restrict the discussion here to low transmission coefficients, $\epsilon_b \ll T_{c0}$. We assume $T - T_{c0} \gg \epsilon_b$. For this case we have $I_s(\varphi) \doteq I_c \sin\varphi$, where

$$eI_c(T)R = \epsilon_b[F_2(T) + F_1^2(T)/\ln(T/T_{c0})], \qquad F_j(T) = 2\pi T \sum_{n\geq 0} f_s^j(\omega_n)/\omega_n. \tag{5'}$$

It follows from (5′) that under the condition $\epsilon_b \ll T_{c0}$ the supercurrent is small:

$I_c(T) \ll I_c(0)$. Also small in this case is the gap, $\Delta_g = (\tilde{\Delta} + \epsilon_b)\cos(\varphi/2) \ll \Delta_0$; the superconducting transition temperature is T_{c0}.

The situation changes substantially when a voltage is applied, because of the deviation of the distribution function $\bar{f}(\epsilon)$ from equilibrium. In discussing this case we consider the region $T > T_{c0}$, and we assume that the following inequalities hold:

$$T_c^h \gg T_{c0} \gg \epsilon_b \tag{6}$$

Under these conditions we can ignore the proximity effect.[1)] In this case the most interesting effects are seen at voltages $V \gg \epsilon_b/e$, at which we can ignore the oscillatory part of the distribution function (and also those of the Green's functions $\hat{G}^{(R,A)}$ and the order parameter). For the steady-state part of $\bar{f}(\epsilon)$ we find the following equation from (1) $(R_1 = R_2)$:

$$\nu_h(\epsilon_+)[n(\epsilon_+) - \bar{f}(\epsilon)] + \nu_h(\epsilon_-)[n(\epsilon_-) - \bar{f}(\epsilon)] = \tau_b I_{ph} \ . \tag{7}$$

Here $\nu_h(\epsilon) = |\epsilon|\theta(|\epsilon| - \Delta_h)/(\epsilon^2 - \Delta_h^2)^{1/2}$ is the density of states of superconductor S_h, $\epsilon_\pm = \epsilon \pm eV/2$, $n(\epsilon) = \tanh(\epsilon/2T)$, $\tau_b = \hbar/\epsilon_b$ is a time scale determined by tunneling processes, and $I_{\rm ph}$ is a collision integral representing collisions with phonons.[5,6] By virtue of conditions (6), we have the following equation for Δ, which we find from the self-consistency equation:[5,6]

$$\int_0^\infty d\epsilon[\bar{f}(\epsilon)\theta(|\epsilon| - \Delta)/(\epsilon^2 - \Delta^2)^{1/2} - n(\epsilon)/\epsilon] = \ln(T/T_{c0}). \tag{8}$$

Solution (7) takes its simplest form in the case in which the time scale for inelastic relaxation satisfies $\tau_{in} \gg \tau_b$. In this case we find

$$\begin{aligned} \bar{f}(\epsilon) = {} & [\nu_h(\epsilon_+)n(\epsilon_+) + \nu_h(\epsilon_-)n(\epsilon_-)]/[\nu_h(\epsilon_+) + \nu_h(\epsilon_-)] \\ & + n(\epsilon)\theta(\Delta_h - |\epsilon_+|)\theta(\Delta_h - |\epsilon_-|). \end{aligned} \tag{9}$$

From (7) and (8) we find an equation for $\delta = \Delta/\Delta_0$. This equation takes a particularly simple form at temperatures $T \ll T_c^h$:

$$[\ln[(u^2 - \delta^2)^{1/2} + |u|] - \tilde{v}(\delta, u)]\theta(|u| - \delta) + \ln(\delta)\theta(\delta - |u|) = 0, \tag{10}$$

where $u = (eV - 2\Delta_h)/2\Delta_0$,

$$\tilde{v}(\delta, u) = \begin{cases} \displaystyle\int_0^{(u^2-\delta^2)^{1/2}} dx \tan(\beta x)/(x^2 - \delta^2)^{1/2}, & u < 0, \\ (\pi/2) - (2u/\delta)\arctan[\delta/(u + (u^2 - \delta^2)^{1/2})], & u > 0, \end{cases}$$

Here $\beta = \Delta_0/2T$. We easily see from (10) that under the condition $|u| < 1$ there exists a V-independent solution $\delta = 1$. There are also solutions (which differ only in the region $u < 0$) which we found numerically and which are shown for three tempera-

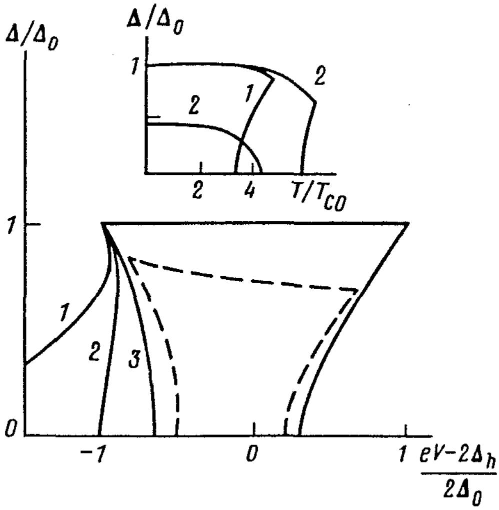

FIG. 1. Order parameter versus the voltage under the conditions $\tau_{in} \gg \tau_b$ and $T_{c0} = 0.1 T_c^h$ for various temperatures. 1—$t = T/T_{c0} = 1$; 2—$t = 1.2$; 3—$t = 2$. The dashed line here shows the dependence corresponding to $t = 2$ in the case $\tau_{\rm in} = \tau_b$. The inset shows the temperature dependence of the gap at two values of $u = (eV - 2\Delta_h(0))/2\Delta_0$. 1) $u = -0.5$; 2—$u = 0.5$.

tures in Fig. 1. The temperature dependence of Δ at a fixed voltage is interesting; it is shown for two values of V in the inset in Fig. 1. We see that these solutions are discontinuous (there is a first-order phase transition) at a certain critical temperature. The temperature region in which the solutions exist is bounded in a critical fashion by temperatures whose V dependence, found numerically for the case $\tau_{in} \gg \tau_b$, is shown in Fig. 2.

Since the $\Delta(V)$ dependence is multivalued, we find some interesting features on

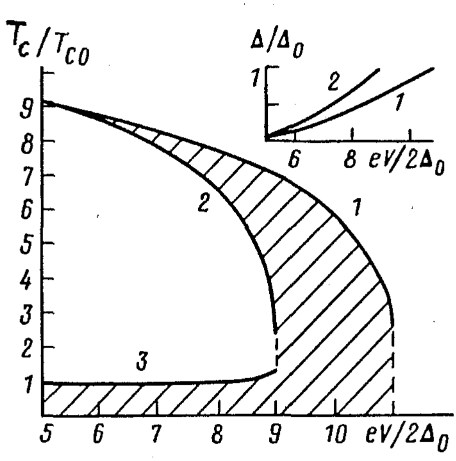

FIG. 2. T_c versus the voltage under the conditions $\tau_{in} \gg \tau_b$ and $T_{c0} = 0.1 T_c^h$. These curves determine the region in which a solution for Δ exists (the hatched region). The inset shows the $\Delta(T_c)$ dependence corresponding to curves 1 and 2; curve 3 corresponds to $\Delta(T_c) = 0$.

the current-voltage characteristic at voltages near $2\Delta_h(T)/e$. From (3) we find the following expression for the I-V characteristic under the condition $eV \gg \epsilon_b$:

$$I(V) = \frac{1}{2eR} \int_{-\infty}^{\infty} d\epsilon [\bar{f}(\epsilon) - n(\epsilon - eV/2)] \nu_h(\epsilon - eV/2) \nu(\epsilon), \tag{11}$$

where $\nu(\epsilon) = |(\epsilon)|\theta(|\epsilon| - \Delta)/(\epsilon^2 - \Delta^2)^{1/2}$. A particularly simple result follows from (11) in the case $\tau_{in} \gg \tau_b$ at $T \ll T_c^h$. In this case, the dependence on $u = (eV - 2\Delta_h(0))/2\Delta_0$ is determined by the expression ($|u| \sim 1$)

$$eI(V)R/\Delta_0 = \sqrt{(T_c^h/T_{c0})\sqrt{u}}[2,132\theta(u-1) + Y(\delta(u)/u)\theta(u - \delta(u))\theta(1-u)], \tag{12}$$

where

$$Y(x) = 2\int_x^1 dy[(1+y)^{1/2} - (1-y)^{1/2}]/(y^2 - x^2)^{1/2}$$

and $\delta(u)$ is a solution of Eq. (10). In the case in which energy relaxation is unimportant at low temperatures (within exponentially small terms), the current is thus zero at $u < 0$. To reach a qualitative understanding of how the energy relaxation affects the value of Δ and the current, it is sufficient to substitute in solution (8) for $\bar{f}(\epsilon)$, found through the use of a simplified expression for the collision integral: $I_{ph} = [\bar{f}(\epsilon) - \tanh(\epsilon/2T)]/\tau_{in}$. Figures 1 and 3 show solutions for Δ and current-voltage characteristics for $\tau_{in} = \tau_b$ and $T = 2T_{c0}$ found as a result of this substitution.

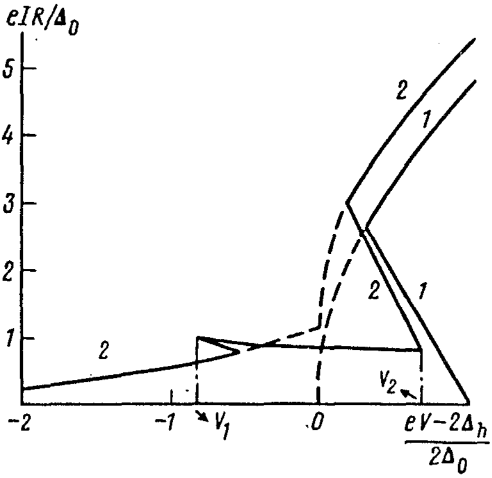

FIG. 3. Current versus voltage near $eV = 2\Delta_h$ under the condition $T = 2T_{c0} = 0.22T_c^h$. 1—$\tau_{in} \gg \tau_b$; 2—$\tau_{in} = \tau_b$. Curve 2, in the given-current regime, corresponds to voltage jumps which occur as the current is raised (or lowered), at $I = I(V_1)[I = I(V_2)]$. The dashed line corresponds to $I(V)$ in the case $\Delta = 0$, i.e., the current-voltage characteristic of an $S_h INIS_h$ structure (where N is a normal metal).

We see that the multivalued nature of the $\Delta(V)$ dependence makes the I-V characteristic multivalued. This function has regions with a negative conductivity $\sigma = dI/dV$, which correspond to solutions for Δ which decrease with increasing V. Curve 2 in Fig. 3 corresponds, in a given-current regime, to a hysteresis and to a voltage drop at $I = I(V_{1,2})$, where $V_{1(2)} = 2(\Delta_h \mp \Delta(V_{1(2)})/e$. The shape found for the current-voltage characteristic corresponds to the case $R_1 = R_2$ under the conditions T_{c0}, $T \ll T_c^h$. The latter inequality was not a strong inequality in the experiments of Blamire *et al.*,[1] and the barriers differed in resistance. For this reason, a singularity was observed on only the current-voltage characteristic, as a step at $V = 2(\Delta_h - \Delta(V))/e$ (in the given-current regime). The temperature dependence of Δ reported in Ref. 1 shows that the gap does have a discontinuity, in agreement with the conclusions reached here. As we have already mentioned, two first-order phase transitions may occur at certain values of V. These transitions must be accompanied by an abrupt change in I as a function of T at a fixed V. It would accordingly be interesting to see an experimental study of this dependence.

In addition to the quasiparticle component of the current, an oscillatory Josephson component of the current flows through the structure (this component is oscillatory with a time-independent V). We will not reproduce the expression for this Josephson component; we will instead simply list the fundamental points. Upon the appearance of a gap in the interval $|eV - 2\Delta_h| < 2\Delta_0$ (at $T > T_{c0}$), there is a sharp increase in the Josephson component of the current. This component is negligible at other voltages. There is also a change in its oscillation frequency, from $\omega_j = 2eV/\hbar$ to half this value (if the barriers are identical). This change could be detected by observing the Josephson steps. At $T > T_{c0} \gg \epsilon_b$ this structure thus amounts to a weak link in which there is essentially no steady-state Josephson effect, but there is a time-varying Josephson effect.

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[1)]The influence of the proximity effect on the properties of various two-barrier structures was studied in Refs. 3, 4, 8, and 9 by the microscopic approach which we are using in the present letter.

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