Generalization of Newman-Penrose dyads in connection with the action integral for supermembranes in an 11-dimensional space

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A twistor-like formulation is proposed for supermembranes in an 11-dimensional space ($D=11$). Spinor harmonics which generalize Newman–Penrose dyads to the $D=11$ case and which are required for a covariant quantization are introduced.

1. The covariant quantization of $D=10$ superstrings and $D=11$ supermembranes is pertinent to the problem of a covariant description of κ symmetry.[1] This problem was recently solved for null superstrings and null supermembrances in $D=4$ (Ref. 2). A crucial aspect of the approach of Ref. 2 is the use of a new twistor-like representation of the action integral of null super-p-branes ($p=0,1,2,3$), which includes the fields of a moving Cartan n-hedron. They are realized by means of commuting spinor variables T_α, O_α—Newman-Penrose dyads[3]—or, equivalently, Lorentz harmonics v_α^-, v_α^+ (Ref. 4). A new twistor-like formulation of the Green-Schwarz superstring was recently proposed[5] on the basis of a generalization of dyads to the $D=10$ case.[6] Since a supermembrane in $D=11$ is an alternative basis for a unified theory of interactions, the problem of a covariant description of these entities is of much interest.[7] In the present letter we propose a twistor-like formulation for the action of supermembranes in $D=11$ on the basis of a generalization of Newman–Penrose dyads (Lorentz harmonics) to the $D=11$ case.

2. Here is the proposed generalization of the twistor-like action functional of supermembranes[5] to the case of extended supersymmetric entities with $p>1$ (super-p-branes) in a D-dimensional space:

$$S_{D,N,p}=-\int d\xi^{p+1}[J^\mu_m\omega^m_\mu-ce]+S^{WZ}_{D,N,p}$$

$$=\int d\xi^{p+1}(\det e^\mu_f)^{-1}[-(\alpha')^{-1/2}e^\mu_f\omega^m_\mu u^{(f)}_m+c]+S^{WZ}_{D,N,p}, \qquad (1)$$

where J^μ_m ($m=0,1,\ldots,D-1$; $\mu=0,\ldots,p$) are the components of the covariant sheet vector current density of the super-p-brane, which is in turn given by

$$J^\mu_m=(\alpha')^{-1/2}e\,e^\mu_f(\xi)\,u^{(f)}_m(\xi),\quad e^{-1}\equiv(\det e^\mu_f),\quad (f=0,\ldots,p). \qquad (2)$$

Here e^μ_f is the inverse sheet "tetrad," c is a dimensionless ("cosmological") constant, and $d\xi^\mu\omega^m_\mu\equiv d\xi^\mu(\partial_\mu x^m-i\partial_\mu\Theta^I C\Gamma^m\Theta^I)$ is the Cartan supersymmetric form for an N-expanded global supersymmetry ($I=1,\ldots,N$). The quantity $S^{WZ}_{D,N,p}$ in (1) is the

Wess–Zumino term for a super-p-brane in D dimensions. The form of this term is well known.[8] Its existence imposes some severe restrictions on the values of the "triad" (D,N,p).

The quantities $u_m^{(f)}$ in (1) and (2) represent a set of $(p+1)$ vectors $u_m^{(n)} \equiv (u_m^{(0)} u_m^{(1)},...,u_m^{(D-1)}) \equiv (u_m^{(f)}(\xi), u_m^{(i)}(\xi))$ of the orthonormal mobile n-hedron which are tangent to the hypersheet:

$$u_m^{(n)} u^{m(k)} = \eta^{(n)(k)} = \mathrm{diag}(1,-1,...,-1). \tag{3}$$

Conditions (3) mean that among the D^2 components of the vectors $u_m^{(n)}$ only $D(D-1)/2$ are independent, so the vectors $u_m^{(f)}$ in action (1) cannot be regarded as independent dynamic variables. The basic idea of the twistor-harmonic approach,[2,4,9] is to introduce spinor Lorentz-harmonic variables,[2,4–6] which parametrize the vectors $u_m^{(n)}$ in accordance with the general law

$$u_m^{(n)} \equiv \frac{1}{2^\nu} U_{\alpha}^{\hat{\alpha}} (C\Gamma_m)^{\alpha\beta} U_{\beta}^{\hat{\beta}} (\Gamma^{(n)} C^{-1})_{\hat{\alpha}\hat{\beta}}. \tag{4}$$

Conditions (3) then hold automatically if the $2^\nu \times 2^\nu$ matrix of Lorentz-harmonic variables $U_{\alpha}^{\hat{\alpha}}$ satisfies the harmonic conditions. The form of these conditions varies with D. Here we have $\nu = [D/2]$ or, alternatively, $\nu = D/2 - 1$ for the case of even D and for Weyl spinors. After representation (4) is substituted into functional (1), we obtain a twistor-like action of a Lorentz-harmonic formulation of super-p-branes.

The harmonic conditions reduce the number of $2^\nu \times 2^\nu$ independent degrees of freedom in $U_{\alpha}^{\hat{\alpha}}$ to the deminsionality of the corresponding Lorentz group $SO(1,D-1)$ (Refs. 2, 4–6). This dimensionality is the same as the number $[D(D-1)/2]$ of independent components of the vectors $u_m^{(n)}$ in (3). Since action (1) is invariant under gauge transformations from the subgroup $SO(1,p) \times SO(D-p-1)$ of the group $SO(1,D-1)$, the number of independent degrees of freedom of the Lorentz harmonics decreases to the dimensionality of the $SO(1,D-1)/SO(1,p)\times SO(D-p-1)$ factor space, which is $(D-p-1)(p+1)$. The total number of independent variables, in addition to the $x^m(\xi)$, from $U_{\alpha}^{\hat{\alpha}}$ and e_f^μ which are contained in action (1) is then $D(p+1)$. It is the same as the number of components $(p+1)$ of the tangents to the world hypersheet of vectors $\partial_\mu x^m(\xi)$ (or ω_μ^m). This circumstance reflects the fact that the vectors $\partial_\mu x^m(\xi)$ (or ω_μ^m) can be expanded in the vectors $u_m^{(f)}$ with exapansion coefficients which depend on e_f^μ, and vice versa. Explicit expressions for these quantities can be found from the equations of motion constructed by varing action (1), as in the case of strings.[10] After these equations are solved, the variables $u_m^{(f)}$ and e_f^μ can be eliminated from the action functional. As a result, the latter assumes the generalized Dirac–Nambu–Goto form (if $S_{D,N,p}^{WZ}$ is ignored).

3. The point of greatest interest is the twistor-harmonic formulation of a supermembrane ($p=2$) moving in 11-dimensional Minkowski space. In this case the spinor Lorentz-harmonic variables parametrize the factor space $SO(1,10)/SO(1,2)\times SO(8)$ and form a 32×32 matrix $U_{\alpha}^{\hat{\alpha}} = (v_{\alpha A}^{a}, v_{\alpha \dot{A} a})$ [$a,b = 1, 2$ are the spinor indices of $SO(1,2)$; $A, B = 1,...,8$; $\dot{A}, \dot{B} = 1,...,8$ are s-spinor and c-spinor indices of $SO(8)$ (Ref.

10); and $\hat{a} = \binom{a}{\dot{a}\,\dot{A}a}$ is the joint spinor index of SO(1,2)×SO(8)]. They are limited by the following harmonic conditions:

$$U^{\hat{a}}_{\hat{\alpha}} C^{\hat{\alpha}\hat{\beta}} U^{\hat{b}}_{\hat{\beta}} = C^{\hat{a}\hat{b}}, \quad U^{\hat{a}}_{\hat{\alpha}} (C\Gamma_{m_1 m_2})^{\hat{\alpha}\hat{\beta}} U^{\hat{b}}_{\hat{\beta}} (\Gamma^{(n)} C^{-1})_{\hat{a}\hat{b}} = 0,$$

$$U^{\hat{a}}_{\hat{\alpha}} (C\Gamma_{m_1 \ldots m_5})^{\hat{\alpha}\hat{\beta}} U^{\hat{b}}_{\hat{\beta}} (\Gamma^{(n)} C^{-1})_{\hat{a}\hat{b}} = 0. \tag{5}$$

These conditions prune $496 + 11 + 462 = 969$ ($= 1024–55$) independent degrees of freedom from the harmonic matrix. As a result, the harmonics $v^{a}_{\hat{\alpha}A}, v_{\hat{\alpha}\dot{A}a}$ carry $55 = \dim SO(1,10)$ independent degrees of freedom,[1)] of which $31 = 3 + 28$ are purely gauge degrees of freedom, by virtue of the requirement of SO(1,2)×SO(8) gauge invariance. The first relation in (5) can be used to construct an inverse harmonic matrix in terms of the same variables, $v^{\hat{a}}_{\hat{\alpha}A}, v_{\hat{\alpha}\dot{A}a}$: $(U^{-1})^{\hat{\beta}}_{b} = (-C^{\hat{\beta}\hat{\alpha}}\epsilon_{ba} v^{a}_{\hat{\alpha}A}, -C^{\hat{\beta}\hat{\alpha}}\epsilon^{ba} v_{\hat{\alpha}\dot{A}a})$. {Here we are using the following representation of the charge conjugation matrix and the γ matrices in $D = 11$:

$$C^{\hat{a}\hat{b}} = -C^{\hat{b}\hat{a}} = \mathrm{diag}(\epsilon^{ab}\delta_{AB}, -\epsilon_{ab}\delta_{\dot{A}\dot{B}}), \quad \Gamma^{(m)} \equiv (\Gamma^{(f)}, \Gamma^{(i)}),$$

$$\Gamma^{(f)} \equiv (\Gamma^{0}, \Gamma^{9}, \Gamma^{10})$$

$$= \mathrm{diag}(\gamma^{(f)}{}_{a}{}^{b}\delta_{AB}, -\gamma^{(f)}{}_{b}{}^{a}\delta_{\dot{A}\dot{B}}),$$

$$\Gamma^{(i)} \equiv (\Gamma^{1}, \ldots, \Gamma^{8}) = \begin{bmatrix} 0 & \epsilon_{ab}\gamma^{(i)}{}_{A\dot{B}} \\ -\epsilon^{ab}\tilde{\gamma}^{(i)}{}_{\dot{A}B} & 0 \end{bmatrix}.$$

Here $\gamma^{(f)}{}_{b}{}^{a}$ are three-dimensional gamma matrices; $\gamma^{(i)}$ are gamma matrices for the SO(8) group;[11] and $\tilde{\gamma}^{(i)}{}_{A\dot{B}} = (\gamma^{(i)}{}_{A\dot{B}})^{T}$.

Proceeding on the basis of these arguments, we write the action in (1) in the form of the following functional for the case of supermembranes ($p = 2$) in $D = 11$:

$$S_{11,1,2} = \int d\xi^{3}\, e[c - \frac{1}{32}(\alpha')^{-1/2} e^{\mu}_{f} \omega^{m}_{\mu} \{v^{a}_{\hat{\alpha}A} v^{c}_{\hat{\beta}A}\epsilon_{bc} + \epsilon^{ac} v_{\hat{\alpha}\dot{A}c} v_{\hat{\beta}\dot{A}b}\}$$

$$\times \gamma^{(f)}{}_{a}{}^{b}(C\Gamma_{m})^{\hat{\alpha}\hat{\beta}}] + S^{WZ}_{11,1,2}. \tag{6}$$

This functional contains the Lorentz harmonics $v^{a}_{\hat{\alpha}A}, v_{\hat{\alpha}\dot{A}a}$, which parametrize the SO(1,10)/SO(1,2)×SO(8) factor space and which are treated as independent dynamic variables, equivalent in rank to x^{m}, Θ, and e^{μ}_{f}. The presence of harmonics in (6), on the one hand, and the twistor-like form of this expression, on the other, lead to a covariant separation of spinor constraints into irreducible constraints of the first and second kinds and thus solve the problem of a covariant description of κ symmetry. On the other hand, this result may facilitate the solution of nonlinear equations of motion of a supermembrane and its covariant quantization, in accordance with the scheme developed in Ref. 2 for the case of null super-p-branes. Evidence in favor of this conclusion comes from experience in the use of the Newman–Penrose dyad to solve nonlinear equations in the theory of black holes[3] and from the existing results on the description and classification of instanton solutions and monopole solutions of the Yang–Mills equations derived by twistor methods (Ref. 12, for example).

[1]In the vicinity of $U=\hat{I}$, the harmonic matrix can be written as $U=\exp(A^{mn}\Gamma_{mn})$, by virtue of (5).

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