Nonperturbative Neveu-Schwarz superstring and Superdiff$S^1/OSp(1|2)$

S. A. Merkulov
Department of Theoretical Problems, Russian Academy of Sciences, 121002, Moscow

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The space of complex structures on the phase space of an open Neveu–Schwarz superstring is isomorphic with respect to the Kähler supermanifold Superdiff$S^1/OSp(1|2)$, not with respect to SuperdiffS^1/S^1, as was asserted in some recent papers. The Ricci tensor of the supermanifold Superdiff$S^1/OSp(1|2)$ is calculated. Nonperturbative vacuum states of the superstring are constructed.

Harari *et al.*[1] and Zhao *et al.*[2] have generalized the nonperturbative boson-string approach of Bowick and Rajeev[3] to the cases of Neveu–Schwarz and Ramond superstrings. A key role is played in this formulation by the space $\mathscr{M}$ of complex structures on the phase space $\Omega M^{d|d}$ of an open superstring. In the present letter we show that in the case of the Neveu–Schwarz superstring a natural complex structure J on $\Omega M^{d|d}$ is invariant under not only the rotation group S^1 but also the larger symmetry group $OSp(1|2)\subset$ SuperdiffS^1. From this result we draw the important conclusion that the space $\mathscr{M}$ is isomorphic with respect to the uniform Kähler supermanifold Superdiff$S^1/OSp(1|2)$, not with respect to SuperdiffS^1/S^1, as was asserted in Refs. 1 and 2 and many subsequent papers.

We denote by M^d a d-dimensional Minkowski space. We denote by $\mathscr{L}M^{d|d}$ the space of all maps $x\oplus\psi:[0,2\pi]\to M^{d|d}\equiv M^d\oplus\Pi M^d$, which satisfy the conditions $x^\mu(0)=x^\mu(2\pi)$ and $\psi^\mu(0)=-\psi^\mu(2\pi)$, where $\mu=0,1,...,d$.

Harari *et al.*[1] showed that the factor space $\Omega M^{d|d}\equiv\mathscr{L}M^{d|d}/M^d$ can be identified with the phase space (or configuration space) of an open (or closed) Neveu–Schwarz superstring.

The vector fields ($m\in Z$, $r\in Z+1/2$)

$$L_m=-i\int d\sigma e^{im\sigma}\left\{\frac{dx^\mu(\sigma)}{d\sigma}\frac{\delta}{\delta x^\mu(\sigma)}+\left[\frac{d\psi^\mu(\sigma)}{d\sigma}+i\frac{m}{2}\psi^\mu(\sigma)\right]\frac{\delta}{\delta\psi^\mu(\sigma)}\right\},$$

$$G_r=\int d\sigma e^{ir\sigma}\left\{\psi^\mu(\sigma)\frac{\delta}{\delta x^\mu(\sigma)}+i\frac{dx^\mu(\sigma)}{d\sigma}\frac{\delta}{\delta\psi^\mu(\sigma)}\right\}\qquad(1)$$

on $\Omega M^{d|d}$ satisfy the commutation relations

$$[L_m,L_n]=(m-n)L_{m+n},\quad [L_m,G_r]=\left(\frac{1}{2}m-r\right)G_{m+r},\quad [G_r,G_s]=2L_{r+s},\qquad(2)$$

and they accordingly realize a representation of the superalgebra SuperdiffS^1.

If $x^\mu(\sigma)$ and $\psi^\mu(\sigma)$ are expanded in Fourier series,

$$x^\mu(\sigma)=\sum_{n\in Z}x_n^\mu e^{in\sigma},\quad \psi^\mu(\sigma)=\sum_{r\in Z+1/2}\psi_r^\mu e^{ir\sigma},$$

the sets mod$\{x_n^\mu,\psi_r^\mu,n\in Z\setminus 0,r\in Z+\frac{1}{2}\}$ form a coordinate system on $\Omega M^{d|d}$. In these coordinates, the generators of (1) are

$$L_m=-\sum_{n\in Z\setminus 0}(n+m)x^\mu_{-n-m}\frac{\partial}{\partial x^\mu_{-n}}-\sum_{s\in Z+1/2}(r+\frac{m}{2})\psi^\mu_{-s-m}\frac{\partial}{\partial\psi^\mu_{-s}},$$

$$G_r=\sum_{n\in Z\setminus 0}\psi^\mu_{-n-r}\frac{\partial}{\partial x^\mu_{-n}}+\sum_{s\in Z+1/2}(r+s)x^\mu_{-s-r}\frac{\partial}{\partial\psi^\mu_{-s}}. \tag{3}$$

We denote by $W=\Sigma_{n\in Z\setminus 0}W_n^\mu(\partial/\partial x_n^\mu)+\Sigma_{r\in Z+1/2}W_r^\mu(\partial/\partial\psi_r^\mu)$ some arbitrary vector field on $\Omega M^{d|d}$. A natural complex structure J on $\Omega M^{d|d}$ is defined by

$$J(W)=-i\sum_{n\in Z}\mathrm{sgn}(n)W_n^\mu\frac{\partial}{\partial x_n^\mu}-i\sum_{r\in Z+1/2}\mathrm{sgn}(r)W_r^\mu\frac{\partial}{\partial\psi_r^\mu}. \tag{4}$$

It can be shown that this complex structure satisfies the relations $J([L_0,W])=[L_0,J(W)]$, $J([G_{-1/2},V])=[G_{-1/2},J(V)]$, $J([G_{+1/2},T])=[G_{+1/2},J(T)]$ for arbitrary vector fields W, V, and T. We thus have

$$\mathcal{L}_{L_0}J=0,\quad \mathcal{L}_{G_{-1/2}}J=0,\quad \mathcal{L}_{G_{+1/2}}J=0,$$

where $\mathcal{L}$ is the Lie derivative along the vector field V. The complex structure is therefore invariant under not only the rotation group S^1 but also the less obvious symmetry group $OSp(1|2)\subset\mathrm{Superdiff}S^1$, which is generated by L_0, $G_{1/2}$, and $G_{-1/2}$. Consequently, the space $\mathcal{M}$ of all complex structures on $\Omega M^{d|d}$ is isomorphic with respect to the supermanifold $\mathrm{Superdiff}S^1/OSp(1|2)$, not with respect to $\mathrm{Superdiff}S^1/S^1$, as was asserted in Refs. 1 and 2.

Let us calculate the Ricci tensor of this supermanifold. We assume that L_m, $m\in Z$, and G_r, $r\in Z+1/2$, are generators of the superalgebra $\mathrm{Superdiff}S^1$. We will use the same symbols to represent the corresponding left-invariant vector fields on $\mathcal{M}$. An arbitrary vector θ which is tangent to $\mathcal{M}$ can be written as the linear combination

$$\theta=\sum_{n\neq\pm1,0}\theta_nL_n+\sum_{r\neq\pm1/2}\theta_rG_r.$$

We define the nearly complex structure $\tilde{J}$ at zero, $0\in\mathcal{M}$, by means of

$$\tilde{J}(\theta)=\sum_{n\in\neq\pm1,0}-i\,\mathrm{sgn}(n)\theta_nL_n+\sum_{r\neq\pm1/2}-i\,\mathrm{sgn}(r)\theta_rG_r. \tag{5}$$

At other points in $\mathcal{M}$, the nearly complex structure is determined by left shifts. It follows[4] from (1) that the structure $\tilde{J}$ is integrable. Consequently, $\mathcal{M}$ is a complex supermanifold. On $\mathcal{M}$ there exists a unique uniform Kähler form, specified by (cf. Refs. 1–3, 5, 6)

$$\omega(L_m, L_n) = a(m^3 - m)\delta_{m,-n}, \quad \omega(L_m, G_r) = \omega(G_r, L_m) = 0,$$

$$\omega(G_r, G_s) = a(4r^2 - 1)\delta_{r,-s},$$

where a is a nonzero parameter; $n \in Z \setminus \{\pm 1, 0\}$, and $r, s \in \{Z + 1/2\} \setminus \{\pm 1/2\}$.

These $(\tilde{J}, \omega)$ form a Kähler structure on $\mathscr{M}$. The Ricci tensor of the corresponding Kähler matrix is found by Freed's method.[7] It is (cf. Refs. 1 and 2)

$$\mathrm{Ric}(L_{-m}, L_n) = -\frac{10}{8}(m^3 - m)\delta_{m,n}, \quad \mathrm{Ric}(G_{-r}, G_s) = -\frac{10}{8}(4r^2 - 1)\delta_{r,s}.$$

Following the geometric quantization method and Refs. 1–3, we find the curvature tensor of the Fok vacuum stratification B above $\mathscr{M}$:

$$F(L_{-m}, L_n) = \frac{d}{8}(m^3 - m)\delta_{m,n}, \quad F(G_{-r}, G_s) = \frac{d}{8}(4r^2 - 1)\delta_{r,s}.$$

The nonperturbative vacuum states of the Neveu-Schwarz superstring are determined in this approach as nontrivial holomorphic and horizontal sections of the tensor product $B \otimes \Gamma$ of stratifications above $\mathscr{M}$, where Γ is a canonical holomorphic stratification. A necessary and sufficient condition for the existence of such sections is the vanishing

$$0 = F(L_{-m}, L_n) + \mathrm{Ric}(L_{-m}, L_n) = \frac{d - 10}{8}(m^3 - m)\delta_{m,n},$$

$$0 = F(G_{-r}, G_s) + \mathrm{Ric}(G_{-r}, G_s) = \frac{d - 10}{8}(4r^2 - 1)\delta_{r,s}$$

of the complete curvature of the stratification $B \otimes \Gamma$. Nonperturbative SuperdiffS^1-invariant vacuum states of a superstring are possible only in $d = 10$ (in total agreement with the usual perturbative methods for describing superstrings[8]).

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