Steady-state stimulated scattering by a grating nonlinearity in a planar nematic liquid crystal

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Forward stimulated scattering by a grating nonlinearity has been achieved in a planar sample of a nematic liquid crystal (5CB) for light with a wavelength $\lambda = 0.63\,\mu$m and a power ~0.03 W. The wavefront of the incident light is reproduced in the scattering. Because of the small grating period, $3\,\mu$m, there is a frequency shift of 24 Hz in the stimulated scattering, and the rise time of the scattering is ~0.06 s.

Because of the pronounced cubic nonlinearity which nematic liquid crystals usually have, the molecules of these crystals can be reoriented with polarized laser light. This reorientation can be of two types: a quasiuniform type, in which case the variations in the orientational deformation of the director have a length scale on the order of the thickness of the sample, and a grating type, in which case the scale of the deformation is set by the spatial variations in the interference pattern of the light field.

In this letter we are reporting a study of the second case. Let us assume that two light waves which differ in polarization (i.e., an *o*-wave and an *e*-wave), and which are coherent with each other, are propagating in the same direction, normal to the director, in a planar sample of a nematic liquid crystal. The polarization state of the resultant field $\vec{E}$ in the nematic then undergoes spatially periodic variations, as shown schematically in Fig. 1a:

$$\vec{E} = \vec{e}_x E_e \exp(ik_e z) + \vec{e}_y E_o \exp(ik_0 z). \tag{1}$$

In this case the distribution of the director $\vec{n}$ is given by

$$\vec{n}(z,t) \approx \vec{e}_x + \theta(z,t)\vec{e}_y, \tag{2}$$

where θ is the angle between the director and the unit vector $\vec{e}_y$. In this situation the dielectric tensor of the nematic acquires an off-diagonal increment which is periodic along z and which causes a rescattering of the *o*- and *e*-waves into each other.

An amplification can be achieved on the basis of this mechanism through a transfer of energy from the wave of one polarization type to the wave of the other polarization type. There can also be a similar amplification starting at the level of the spontaneous noise, i.e., a stimulated scattering.

In the linear approximation, the equation for the director of the nematic is

$$\eta \frac{\partial \theta}{\partial t} + K_{22}\frac{\partial^2 \theta}{\partial z^2} = \frac{\epsilon_a}{16\pi}\left(E_o^* E_e e^{iqz - i\Omega t} + E_e^* E_o e^{-iqz + i\Omega t}\right), \tag{3}$$

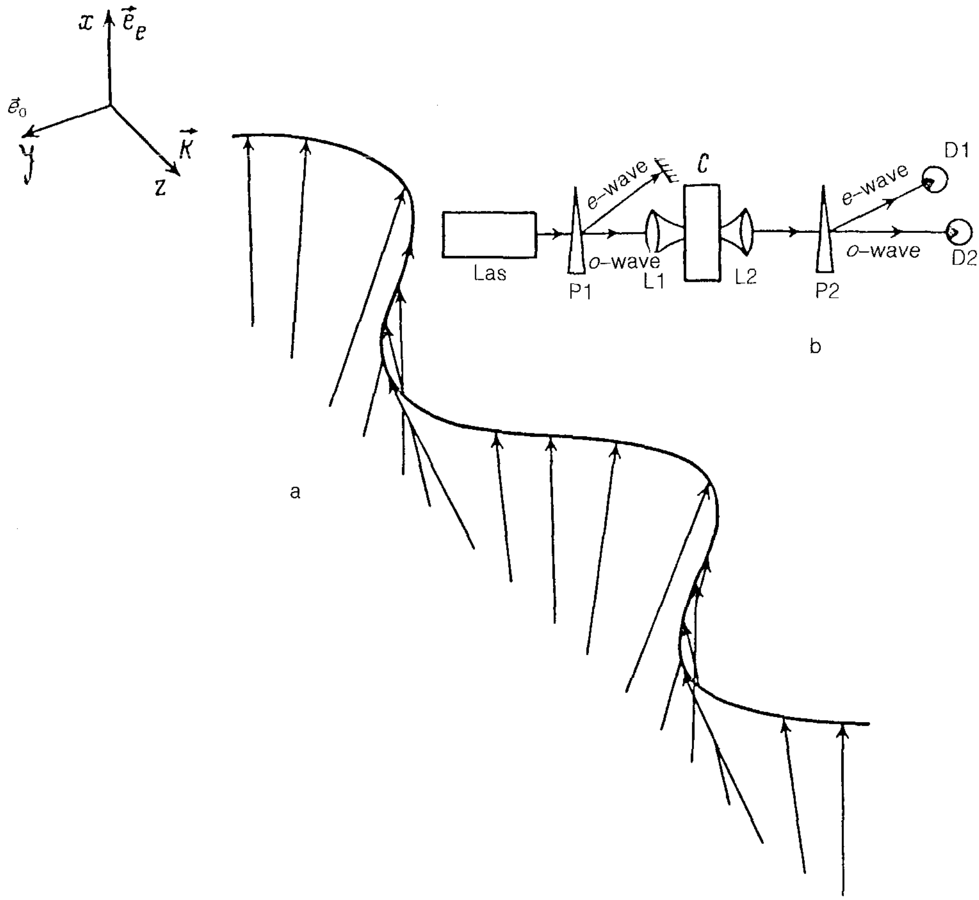

FIG. 1. a: Typical pattern of the periodic perturbations of the director in the planar nematic liquid crystal. This pattern is produced by an interference of the *o*- and *e*-waves. b: Experimental layout. Las—He–Ne laser; P1, P2—polarization wedges; L1, L2—entrance and exit lenses; C—cell with the nematic liquid crystal; D1, D2—photodiode detectors.

where $|\vec{q}| = (\omega/c)(n_e - n_o)$ is the wave vector of the director deformation grating written in the medium, $\epsilon_a = n_e^2 - n_o^2$ is the anisotropy of the dielectric constant of the nematic at the optical frequency ω, $\Omega = \omega_o - \omega_e$ is the frequency shift, K_{22} is the Frank constant, and η is the orientational viscosity.

In the steady state we have

$$\theta(z,t) = \frac{\epsilon_a}{16\pi K_{22} q^2} \left(\frac{E_0^* E_e e^{iqz - i\Omega t}}{1 + i\Omega/\Gamma} + \mathrm{c.c} \right), \tag{4}$$

where $\Gamma = K_{22} q^2/\eta$ is the width of the gain line. The simplified equation for the *e*-polarization signal is then[1]

$$\frac{dE_e}{dz}=\left(\frac{g}{2}+i\delta k\right)E;\quad g+2i\delta k=G|E|^2\cdot 2\frac{\Omega+i\Gamma}{\Omega^2+\Gamma^2}. \tag{5}$$

Here g is the gain (the intensity gain, in units of reciprocal centimeters), G is the gain constant at the point $\Omega=\Gamma$, and δk is the correction to the signal wave vector for mutual focusing. If the power density $|E|^2$ is expressed in units of watts per square centimeters, the dimensions of G are centimeters per watt.

So far, only a time-dependent stimulated scattering has been achieved on the basis of this nonlinearity in the geometry presented above.[2] A steady-state stimulated scattering has been achieved only in the case of a small angle between the director of the nematic and the wave vector of the pump light (the homeotropic orientation of the director). For this reason, the period of the deformation of the director was comparatively large.[3]

In the present letter we are reporting the experimental achievement of steady-state stimulated scattering by a grating nonlinearity in a nematic liquid crystal in a process involving deformation structures with a record-short period (the geometry with a planar orientation of the director).

The experimental layout is shown in Fig. 1b. Linearly polarized light is generated by an LGN-215 He–Ne laser ($\lambda=0.63$ μm) with a power of 0.05 W. This light is focused by a nonabberational lens with a focal length $f=2$ cm into a planar cell 1 mm thick holding the nematic liquid crystal 5CB. The light enters along the normal to the surface of the cell. In this geometry, the laser light is the *o*-wave for the nematic. In addition to the ordinary linear scattering of the ordinary wave into the extraordinary wave by thermal fluctuations of the director (this scattering amounted to ~15–20% of the incident power), we observed a nonlinear stimulated scattering, also from the *o*-wave into the *e*-wave, in a process which clearly involved a threshold. The threshold power of the light incident on the cell was $P_{\text{thr}}\geqslant 5$ mW [the diameter of the focus of the laser beam in the cell with the nematic was $\mathrm{FWe}^{-2}M\sim 10$ μm; the calculated length of the focus was Δz (FWHM) ~500 μm]. We observed a stimulated scattering with reproduction of the wavefront of the pump light. The maximum nonlinear pumping coefficient (R) for the pumping from the *o*-wave into the *e*-wave reached ~50% (further increases were prevented by the limitations of the laser and by the incipient saturation).

The frequency shift between the reference wave and the scattered wave was measured by both indirect and direct methods.

In the first method, a polarizer was placed at the exit from the cell, oriented at an angle of 45° with respect to the polarization of the scattered wave. It was thus possible to observe a traveling interference pattern. An oscilloscope was used to detect the characteristic beats due to the frequency difference between the signals being mixed. The period of these beats was a single-valued function of the frequency shift (Ω) between the reference and scattered waves. In addition to measuring the frequency shift Ω on the basis of the beat oscilloscope traces, we used a computer to calculate Fourier transforms and correlation functions of these traces. Analysis of the beats, their Fourier transforms, and their correlation functions led to the conclusion that the frequency shift in this situation was $\Omega/2\pi\approx 25$ Hz.

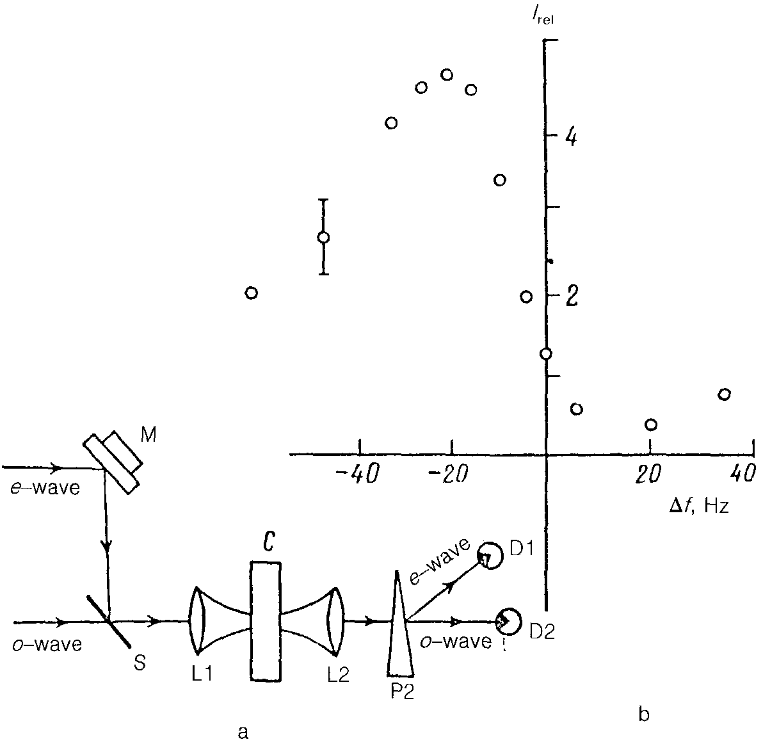

FIG. 2. a: Experimental layout for recording the lineshape of the weak-signal gain. M—Mirror on piezoelectric; S—beam splitter. b: Intensity of the scattered light versus the frequency shift of the weak signal.

In the second method (Fig. 2a), we recorded the lineshape of the weak signal amplified in the pump field. We used a sawtooth voltage generator and a mirror on a piezoelectric ceramic to shift the frequency of the amplified signal by an amount from 0 to 60 Hz, in both the Stokes and anti-Stokes directions. Figure 2b shows the intensity of the scattered light as a function of the frequency shift of the signal.

The maximum amplification of the scattered wave occurred at a frequency shifted 24 ± 5 Hz in the Stokes direction from the reference laser frequency. The theory predicts a frequency shift $\Omega = \Gamma = K_{22}q^2/\eta = 2\pi$ (27 Hz) for the signal in the case of a 5CB crystal at 20 °C. Here we used the values $K_{22} = 5 \times 10^{-7}$ dyn, $\eta \simeq 1.2$ P, and $q \simeq 2 \times 10^4$ cm^{-1} (the period of the grating which was written was $\Lambda = 2\pi/q \approx 3$ μm). We see that the results found by each of these methods agree well with the theory.

The intensity amplification per pass of the signal is described by a factor $\exp(\int g_{\rm eff} dz)$. Allowing for the averaging of the Gaussian signal and pump beams over the cross section after the integration over the entire length of the caustic, which is completely immersed in the medium, we find

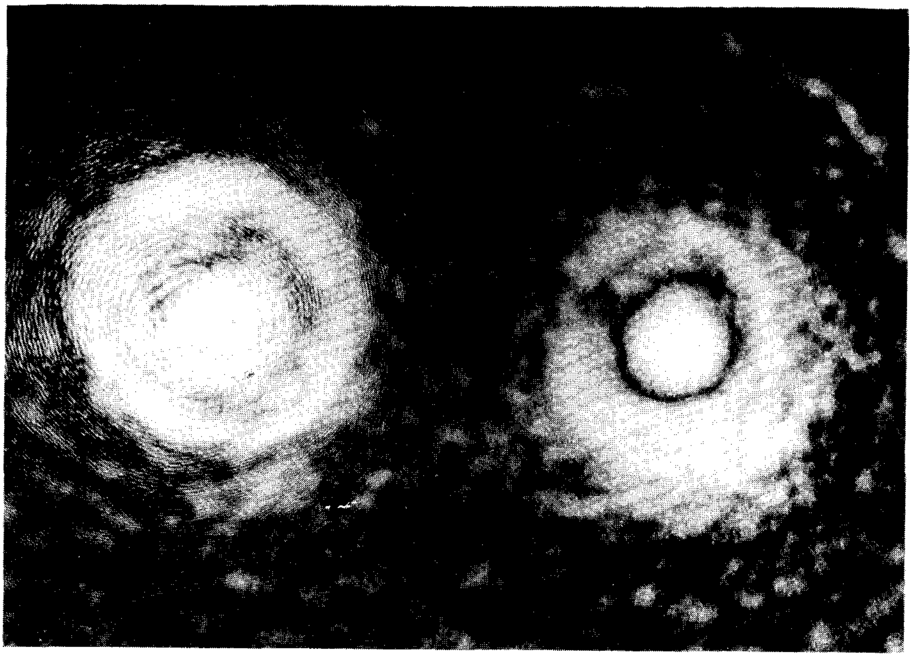

FIG. 3. Mutual focusing of *o*- and *e*-waves by the grating nonlinearity.

$$\int_{-\infty}^{+\infty} g_{\mathrm{eff}}\, dz = \frac{8\pi^2}{\lambda_0 c} GP_0, \tag{6}$$

where P_0 is the power of the incident light, and c and λ_0 are the velocity and wavelength of the light in vacuum.

For this type of degenerate scattering, the theoretical estimate of the gain is $\sim e^{20}$. Analysis of the experimental results yields a gain $\sim e^1$–e^3. Such a difference is possible because the theory ignores the extinction of the *o*-wave in this thick medium (the extinction coefficient for the *o*-wave in 5CB is $\sim 8\ \mathrm{cm}^{-1}$). It is possible that other factors which act to reduce the gain in the nematic are also at work here.

Note also that the measured rise time of the stimulated-scattering process is much longer than would be expected. It is $\sim 6\times 10^{-2}$ s, in contrast with the theoretical prediction $\tau \sim \Gamma^{-1} \approx 6\times 10^{-3}$ s. Again, the reason for this difference is not obvious to us. It is possible that further experiments in this direction will provide answers to these questions.

When an *e*-wave of about the same power was applied along with the *o*-wave to the same cell with the nematic liquid crystal, we observed ring-shaped formations in both the *o*-polarization and the *e*-polarization in the far zone behind the cell (Fig. 3). This effect corresponds to a mutual focusing of these waves by the grating nonlinearity.

In summary, a steady-state stimulated scattering of light in a nematic liquid crystal, by gratings with a very short period, $\Lambda \approx 3\ \mu\text{m}$, has been observed in these experiments. A mutual focusing of *o*-waves and *e*-waves as a result of the same nonlinearity has also been observed.

[1] B. Ya. Zel'dovich and N. B. Tabirin, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 510 (1979) [JETP Lett. **29**, 465 (1979)].

[2] B. Ya. Zel'dovich, S. K. Merzlikin, N. F. Pilipetskiĭ, and A. V. Sukhov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 418 (1985) [JETP Lett. **41**, 514 (1985)].

[3] B. Ya. Zel'dovich, S. K. Merzlikin, and N. F. Pilipetskiĭ, Dokl. Akad, Nauk SSSR **273**, 1116 (1983) [Sov. Phys. Dokl. **28**, 1038 (1983)].

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