Suppression of hot-electron fluctuations in semiconductors under size-effect conditions

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A theory is derived for the fluctuations of hot electrons in a spatially inhomogeneous semiconductor plasma under conditions of a size effect over the energy relaxation length L_ε of the hot electrons. The field dependence of the spectral density of the fluctuations in the electron temperature and that of the current are calculated. The fluctuation spectrum depends on the sample thickness $2d$. It has several nontrivial features. At $2d < L_\varepsilon$, fluctuations at frequencies $\omega \ll \tau_\varepsilon^{-1}$ are suppressed.

The usual consequences of the physics of fluctuation phenomena in a nonequilibrium semiconductor plasma are a growth of the fluctuations (the noise temperature, the spectral density, and the correlation functions) upon an increase in the electric field, because of an increase in the average energy of the carriers and the independence of the intensity from the dimensions of the crystal (except in the trivial case $\sim 1/V$, where V is the volume of the crystal). The growth of fluctuations in strong electric fields and the role played by the factor $1/V$ in nonequilibrium systems of small volume limit the practical applications of semiconductors with submicron dimensions.

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Kinetic phenomena in crystals with small dimensions, whose thickness $2d$ is comparable to the characteristic diffusion length $L=\sqrt{D\tau}$ (D is the diffusion coefficient, and τ is the relaxation time of the nonequilibrium carriers)—classical size effects over a distance L (Ref. 1)—have been studied in detail both experimentally and theoretically. A theory for the fluctuations of hot electrons has been derived without consideration of the effect of the crystal boundaries, in the case in which the average characteristics of the electron gas are spatially homogeneous, and there are only bulk sources of fluctuations.[2] The literature reveals no theoretical work on the fluctuations of hot electrons in an inhomogeneous electron gas under size-effect conditions.

In this letter we are taking a first theoretical look at the fluctuations of hot electrons in a spatially inhomogeneous gas under conditions corresponding to a size effect over the energy relaxation length[3] $L=L_\varepsilon$, with $\tau=\tau_e$. For fluctuations of the electron temperature, we find an exact analytic solution of an equation with Langevin sources, and we derive a nontrivial dependence of the fluctuation spectrum on the crystal thickness. We show that under the condition $2d\lesssim L_\varepsilon$ the fluctuations are suppressed to a significant extent at frequencies

$$\omega\tau_\epsilon, \quad \omega D/(2d)^2 \ll 1. \tag{1}$$

The lower limit on the frequency range considered is determined by the time scales of the generation and recombination processes.

We assume that an external electric field $\vec{E}$ is applied along the x axis to a homogeneous sample of thickness $2d$ in the y direction. We assume that all quantities depend on y and that the electron gas remains homogeneous in the xz plane as the heating occurs. Under the typical conditions

$$\tau_p \ll \tau_{ee} \ll \tau_\epsilon, \quad l_p, l_D \ll 2d, L_\epsilon \tag{2}$$

the steady-state distribution of electrons with respect to energy ε is Maxwellian, $F_0(\varepsilon,T)\sim\exp(-\varepsilon/k_0T(y))$, and the fluctuation in this function is found from the Boltzmann–Langevin kinetic equation. It is directly related to the fluctuation $\delta T(y,t)$, in the temperature of the heated electrons, $T(y)$:$\delta F_0=[\partial F_0(\varepsilon,T)/\partial T]\delta T$, where τ_p is a typical pulse duration, l_p is a typical spatial length of the pulses, τ_{ee} is the time scale of electron–electron collisions, l_D is the Debye length, and t is the time. The fluctuation of the anisotropic part of the distribution function, δF_1, is determined by the fluctuation δF_0 and by the anisotropic part of the Langevin source in the standard way.[2]

The energy loss of the carriers at the surfaces $y=\pm d$ limits the heating and causes a dependence on the transverse coordinate of the quantities $T(y)$ and $\delta T(y,t)$. A transverse steady-state electric field and a transverse fluctuating electric field arise in the sample. In addition to the bulk sources of fluctuations, surface sources arise.

We can write a continuity equation for the Fourier transform of the fluctuations, $\delta T(y,\omega)$, which follows from the Boltzmann–Langevin kinetic equation:

$$-i\omega\frac{3}{2}n\delta T(y,\omega)+\hat{\delta}_T Z(T)=\tilde{U}(y,\omega)-eE\tilde{I}_x(y,\omega)-\frac{d\tilde{R}_y(y,\omega)}{dy}\equiv\tilde{f}(y,\omega),$$

$$Z(T)=\frac{dq_y}{dy}-\sigma(T)E^2+P(T),\quad \tilde{\vec{R}}=\tilde{\vec{Q}}-\frac{d(\sigma T^{5/2})/dT}{\sigma T^{1/2}}\tilde{\vec{I}} \tag{3}$$

We can also write boundary conditions for the fluctuating energy flux, $\vec{\delta}\vec{q}$:

$$\delta q_y(y=\pm d,\omega)=\mp\tilde{U}(y=\pm d,\omega)\pm n\left\{\delta T(y,\omega)\frac{d}{dT}[(T-T_0)S(T)]\right\}_{y=\pm d}, \tag{4}$$

where n is the electron density, $\sigma(T)$ is the conductivity, $P(T)$ is the power transferred from the electrons to the lattice (a heat reservoir with a temperature T_0), and $\delta T=\delta T(d/dT)$. The steady-state and fluctuating energy fluxes are given by the expressions

$$q_y=-\left(\frac{E_y}{eT}+\frac{1}{e^2}\frac{d}{dy}\right)T^{1/2}\frac{d}{dT}(\sigma T^{5/2}),\quad \delta q_y(y,\omega)=\hat{\delta}_T q_y+R_y(y,\omega). \tag{5}$$

The component $E_y(y)$ of the steady-state field and its fluctuating increment $\delta E_y(y,t)$ are found in a self-consistent way with the help of condition (2) and the condition that there is no transverse current: $j_y=0$, $\delta j_y=0$.

The correlation functions of the Langevin sources of fluctuations in the energy, $\tilde{U}(\vec{r},t)$, in the electron flux density, $\tilde{\vec{I}}(\vec{r},t)$, and in the electron energy flux density, $\tilde{\vec{Q}}(\vec{r},t)$, on the right side of Eq. (3) were calculated in Ref. 4. They are δ-correlated. The correlation function of the surface sources of energy fluctuations is expressed in terms of the rate of surface relaxation of the electron energy,[3] $S^{\pm}$:

$$(\tilde{U}\tilde{U})^{\pm}_{\omega}=\frac{4dnT_0T^{\pm}}{V}S^{\pm}. \tag{6}$$

By virtue of condition (2), the expressions for the correlation functions of the surface sources, $\tilde{\vec{I}}$ and $\tilde{\vec{Q}}$, are determined by their bulk values.

For low-frequency fluctuations, in the sense of condition (1), the first term on the left side of Eq. (3) can be ignored. A solution can then be found by the following procedure. Obviously, one particular solution of the homogeneous equation $\hat{\delta}_T Z(T)=0$ is the function $\delta T_1(y,\omega)=dT(y)/dy$, where $T(y)$ is the solution of the steady-state problem

$$Z[T(y)]=0,\quad q_y(y=\pm d)=\pm n(T^{\pm}-T_0)S^{\pm}. \tag{7}$$

This solution can be expressed in quadrature. Using $\delta T_1(y,\omega)$, we find a second particular solution,

$$\delta T_2(y,\omega)=\delta T_1(y,\omega)\int_{-d}^{y}\frac{\exp(-\int^{y'}a\,dy'')}{(\delta T_1)^2}dy'.$$

We can thus find a general solution of Eq. (3):

$$\delta T(y,\omega) = \frac{1}{2n}\left\{\delta T_1 \int_{C_1}^{y} \frac{\delta T_2 \tilde{f}}{D(T)\Delta} dy' - \delta T_2 \int_{C_2}^{y} \frac{\delta T_1 \tilde{f}}{D(T)\Delta} dy'\right\}, \tag{8}$$

where

$$a = \frac{2}{D}\frac{dD(T)}{dy}, \quad \Delta = \delta T_1 (\delta T_2)'_y - \delta T_2 (\delta T_1)'_y.$$

The constants $C_{1,2}$ are found from the boundary conditions, which take a particularly simple form in the case of strong surface scattering, $S^{\pm} \gg D/L_\varepsilon$:

$$T^{\pm} = T_0, \quad (\delta T)^{\pm} = 0. \tag{9}$$

The expressions derived here give, in analytic form, the exact solution of the problem of fluctuations in an inhomogeneous electron gas of hot carriers in the frequency region defined by condition (1).

Let us examine the spectral density of the fluctuations in the electron temperature, $(\delta T \delta T)_\omega$, and in the current density, $(\delta j_i \delta j_k)_\omega$, as a function of E and d. A

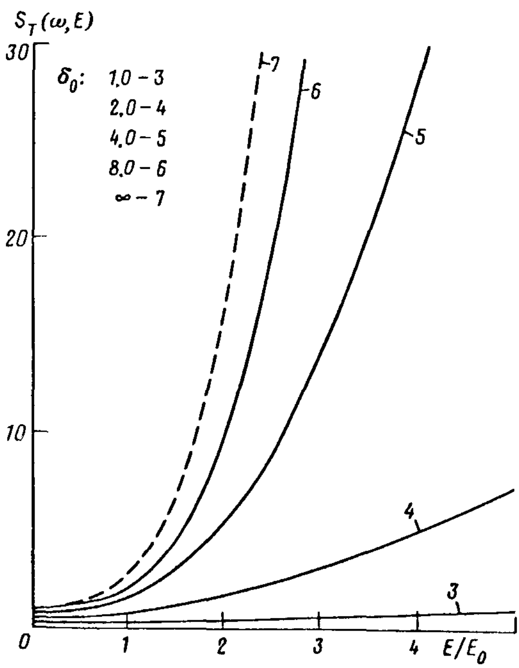

FIG. 1. Field dependence of the spectral density of fluctuations in the electron temperature for various values of δ_0. Here $S_T(\omega,E) = (\delta T\delta T)_\omega(E)/(\delta T\delta T)_\omega^\infty(0)$, where $\delta_0 = d/L_\varepsilon$, and $E_0 = k_0 T_0/eL_\varepsilon$.

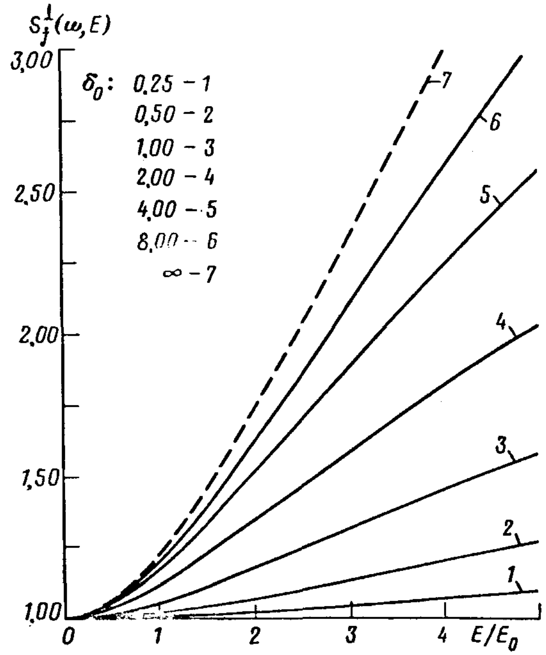

FIG. 2. Field dependence of the spectral density of the fluctuations in the current for various values of δ_0. Here $S_j^{\perp}(\omega,E) = (\delta j_z \delta j_z)_\omega (E)/(\delta j_z \delta j_z)_\omega^\infty (0)$.

fluctuation in the current density, δj_i, is expressed in terms of the fluctuation δT and the Langevin source $\tilde{I}$: $\delta j_i(y,\omega) = (\partial\sigma/\partial T)E_i \delta T(y,\omega) - e\tilde{I}_i(y,\omega)$.

Figures 1–3 show the field dependence of the spectral density of fluctuations for various sample thicknesses. These results were calculated from the expressions derived here, under boundary conditions (9). Calculations were carried out for various combinations of scattering mechanisms. The qualitative picture of the effects remains unchanged. As an illustration here, as in Ref. 3, we assume that the electron momentum is scattered primarily by acoustic phonons, while the quasielastic scattering of the electron energy is a scattering by deformation optical phonons. The spectral density of the fluctuations is calculated as the average over the thickness of the crystal. The index ∞ means the spectral density of the fluctuations in the case in which the boundaries of the crystal have no effect. We see that low-frequency fluctuations of hot electrons have some nontrivial features in bounded semiconductors ($2d \lesssim L_\varepsilon$) with a cooled surface. The most interesting of these features is that these fluctuations are significantly suppressed in comparison with those in a bulk sample (a thin crystal with $S^{\pm} = 0$). This suppression is by an order of magnitude for the case of $S_T(\omega,E)$ and by a factor of several units for $S_j(\omega,E)$. The difference in the behavior of $S_j^{\perp}(\omega,E)$ and $S_j''(\omega,E)$, and also the nonmonotonic d dependence of $S_j''(\omega,E)$ at a fixed value of E in the strong-field region stem from a convective contribution of fluctuations to these quantities. The occurrence of an additional scattering by the surface does not reduce the

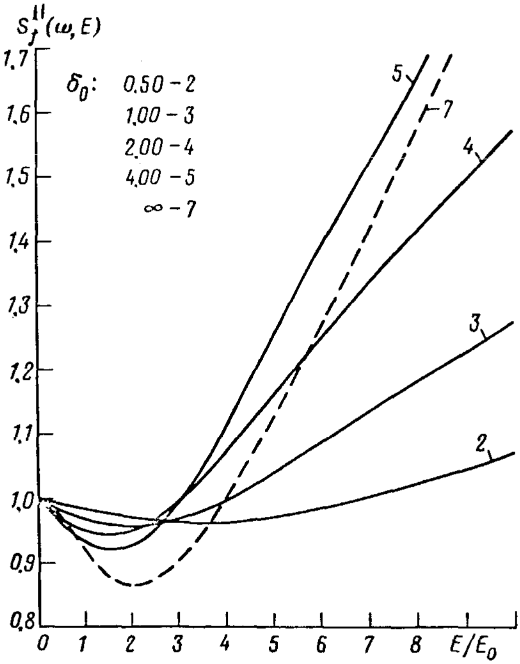

FIG. 3. Field dependence of the spectral density of the fluctuations in the current for various values of δ_0. Here $S_j^{\parallel}(\omega,E) = (\delta j_z \delta j_z)_\omega (E)/(\delta j_z \delta j_z)_\omega^\infty (0)$.

integral intensity of the fluctuations: A redistribution of fluctuations over the spectrum arises under the condition $2d \lesssim L_\varepsilon$. The fluctuations decrease in the low-frequency region and increase at $\omega \approx D/(2d)^2$. The suppression of the fluctuations results in an improvement in the signal-to-noise ratio, which is determined by the parameter $\delta = j_x/\Delta\omega\sqrt{(\delta j_x \delta j_x)\omega}$. In bulk samples $(2d/L_\varepsilon \gg 1)$ the current density reaches saturation in strong fields: $(\delta j_x \delta j_x)_\omega^\infty \sim E$ and $\delta \sim E^{-1/2}$. In other words, the current density decreases with increasing E. For thin crystals $(2d/L_\varepsilon \ll 1)$, Ohm's law, $j_x \propto E$, holds, and the suppression of the fluctuations is $(\delta j_x \delta j_x)_\omega \sim E^{1/2}$. The effect is to increase the signal-to-noise ratio, $\delta \sim E^{3/4}$.

This suppression of low-frequency fluctuations of hot electrons should be seen experimentally under the conditions which have been used in studies of the size effect over a distance L_ε. According to Ref. 5, in n-type Si these conditions correspond to the typical parameter values $n = 10^{14}$ cm^{-3}, a mobility $\mu \approx 10^4$ cm^2/(V·s), $S^\pm \approx 10^6$ cm/s, and $2d \lesssim L_\varepsilon \approx 5 \times 10^{-4}$ cm at $T_0 = 77$ K and $E \approx (1\text{–}4) \times 10^3$ V/cm $(\vec{E} \parallel [100])$.

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