Fluctuational nonlocal diamagnetic response near the superconducting transition temperature

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Expressions for the fluctuational component of the static value of the kernel $Q(k)$ are derived for a normal metal and for a superconductor near T_c. These components depend on the wave vector. Certain consequences of the nonlocal nature of the fluctuational diamagnetic response are discussed.

The temperature dependence of the diamagnetic susceptibility of a normal metal near the superconducting transition temperature T_c is known to be determined primarily by the contribution from fluctuations in the superconducting order parameter. The theoretical and experimental work on this contribution, which has been carried

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out to date, has been restricted to the case in which the relationship between the induced magnetic moment and the applied magnetic field can be regarded as local.[1] However, the spatial dispersion of the fluctuational diamagnetism has a length scale $\xi(T)$, as is shown below. Because of the large macroscopic value of the correlation radius for the superconducting fluctuations, $\xi(T)$, spatial dispersion becomes important even for a magnetic field whose spatial variation is smooth at the macroscopic level. In the superconductor, at $T<T_c$, the fluctuational correction to the relationship between the current and the field near T_c is also nonlocal, with a length scale on the order of $\xi(T)$ serving as a measure of the deviation from a local situation. Below we derive expressions for the fluctuational component of the static value of the kernel $Q(k)$ for the normal phase and for the superconducting phase near T_c. These expressions depend on the wave vector. This kernel relates the spatial Fourier components of the current in the field in the approximation linear in the field. We examine certain consequences of the nonlocal nature of the diamagnetic response.

Let us average the expression for the superconducting electric current over the fluctuations in the order parameter, using the ordinary Ginzburg–Landau functional as the effective Hamiltonian:

$$F=\int dV\left[a|\psi(\vec r)|^2+\frac{b}{2}|\psi(\vec r)|^4+\frac{1}{4m}\left|\left(\vec\nabla-\frac{2ie}{c}\vec A(\vec r)\right)\psi(\vec r)\right|^2\right]. \qquad (1)$$

Under the gauge condition $\operatorname{div}\vec A=0$, we have the following expression for the average current, in the approximation linear in the field:

$$\vec j(\vec k)=-Q(k)\vec A(\vec k). \qquad (2)$$

In the Gaussian approximation, we find an explicit expression for the quantity $Q(k)-Q(0)$; for $Q(0)$ we find a formula which contains an integral which diverges at large momentum. In the case $T>T_c$, in which we are dealing with a normal metal, we should have $Q(0)=0$ by virtue of gradient invariance. This conclusion is of course verifiable by a microscopic calculation. Above T_c, it is convenient to replace $Q(k)$ by the magnetic susceptibility $\chi(k)=-Q(k)/[ck^2+4\pi Q(k)]\simeq -Q(k)/ck^2$. For the latter we find

$$\chi(k)=\chi_0 f\left(\frac{1}{2}k\xi(T)\right)=\frac{3\chi_0}{k\xi(T)}\left[\left(1+\frac{4}{k^2\xi^2(T)}\right)\arctan\left(\frac{k\xi(T)}{2}\right)-\frac{2}{k\xi(T)}\right]. \qquad (3)$$

Here $\xi(T)=1\sqrt{4m|a|}$ is a temperature-dependent coherence length, and $\chi_0=-e^2T_c\xi(T)/6\pi c^2$ is the well-known expression for the fluctuational diamagnetic susceptibility in a uniform field.[2,3] In the limiting cases of small and large values of $k\xi(T)$, we find from (3)

$$\chi(k)=\chi_0\left(1-\frac{k^2\xi^2(T)}{20}\right),\qquad k\xi(T)\ll 1;$$

$$\chi(k)=\frac{3\pi\chi_0}{2k\xi(T)}\left(1-\frac{8}{\pi k\xi(T)}\right),\qquad k\xi(T)\gg 1. \qquad (4)$$

According to (3), the nonlocal kernel which relates the fluctuational magnetization $\vec{M}$ to the magnetic field $\vec{H}$ in coordinate space is exactly the same as the expression proposed by Pippard for the nonlocal relationship between the current $\vec{j}$ and the vector $\vec{A}$ in superconductors (Ref. 4; see also Ref. 5):

$$\vec{M}(\vec{r}) = \frac{3\chi_0}{2\pi\xi(T)} \int \frac{\vec{R}(\vec{R}\cdot\vec{H}(\vec{r}'))}{R^4} e^{-2R/\xi(T)} d\vec{r}'; \quad \vec{R} = \vec{r} - \vec{r}'. \tag{5}$$

An important point is that the length scale of the deviation from a local situation in (5) is $\xi(T) \gg \xi(T=0) \equiv \xi_0$.

Near T_c, an external magnetic field H parallel to the plane boundary of a bulk normal metal induces a magnetization $M_0 = \chi_0 H$ in the interior of the metal. The spatial dispersion of the magnetic susceptibility in (3) governs the profile of the magnetization M over a length scale $\sim\xi(T)$; it varies from zero at the boundary of the metal to M_0 in the interior of the metal. If the sample occupies the half-space $x > 0$, we find the following expression for the magnetization $M(x) = [B(x) - H]/4\pi$ under the condition of specular reflection:

$$M(x) = M_0 \left\{1 - e^{-2x/\xi} + \frac{x}{2\xi}\left[\left(\frac{2x}{\xi} - 1\right)e^{-2x/\xi} + 2\left(\frac{2x^2}{\xi^2} - 3\right)\mathrm{Ei}\left(-\frac{2x}{\xi}\right)\right]\right\}. \tag{6}$$

Here $\mathrm{Ei}(-x) = \int_{-\infty}^{-x} dt e^t/t$. For the case $x \ll \xi/2$, we find from (6)

$$M(x) = \frac{3M_0 x}{\xi(T)} \ln\left(\frac{\xi(T)}{2,16x}\right) \tag{7}$$

In the region $x \gg \xi/2$ we find from (6)

$$M(x) = M_0\left(1 - \frac{3\xi^2(T)}{4x^2} e^{-2x/\xi(T)}\right) \tag{8}$$

Figure 1 shows the profile of $M(x)/M_0$ along the dimensionless coordinate $2x/\xi(T)$.

In the superconductor, at $T < T_c$, the length scale of the nonlocal coupling of the main part (nonfluctuating part) of the supercurrent with the field is known to be determined over the entire temperature range by the coherence length ξ_0 at absolute zero. Near T_c we have $\xi_0 \ll \xi(T)$. In accordance with the approximation under which we can use the Ginzburg–Landau theory, we ignore deviations from a local situation with a length scale ξ_0. The fluctuational contribution to the screening current amounts to only a small correction to the main supercurrent in the Gaussian approximation. The fluctuational correction to the value of $Q(0)$ was recently found by Buzdin and Vuyichit.[6] Fluctuations in the order parameter with respect to the uniform superconducting state are considered here. Incorporating the wave-vector dependence of the fluctuational contribution, we find the following expression in the Gaussian approximation:

$$Q(k) = \frac{c}{4\pi}\lambda_L^{-2}\left[1 + \frac{1}{3}\sqrt{\frac{Gi}{t}}\left(1 + \frac{k^2\xi^2(T)}{2}\right) f\left(\frac{1}{\sqrt{2}}k\xi(T)\right)\right]. \tag{9}$$

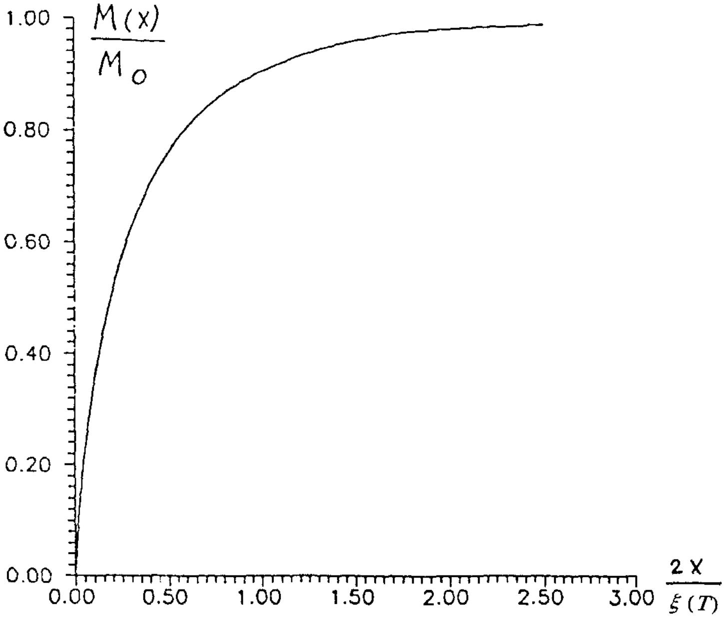

FIG. 1.

Here $t = (T_c - T)/T_c$, and $\lambda_L = (mc^2 b/8\pi e^2|a|)^{1/2}$ is the London depth if fluctuations in the order parameter are ignored. The Ginzburg number is $Gi = 2T_c m^3 b^2/\pi^2\alpha$, where $a = \alpha(T - T_c)$. The function $f(x)$ is defined in (3).

In the case $k\xi(T) \ll 1$, which corresponds to large values of the Ginzburg–Landau parameter, $\kappa \gg 1$, we find from (9)

$$Q(k) = \frac{c}{4\pi}\lambda_L^{-2}\left[1 + \frac{1}{3}\sqrt{\frac{Gi}{t}}\left(1 + \frac{2}{5}k^2\xi^2(T)\right)\right]. \tag{10}$$

We can thus find the fluctuational correction to the penetration depth:

$$\lambda^{-2} = \lambda_L^{-2}\left[1 + \frac{1}{3}\sqrt{\frac{Gi}{t}}\left(1 + \frac{2}{5\kappa^2}\right)\right], \quad \kappa \gg 1. \tag{11}$$

Expression (11) differs from the corresponding result in Ref. 6 in that it also incorporates the small correction $\propto \kappa^{-2}$ due to the nonlocal nature of the response.

Under the condition $k\xi(T) \gg 1$, which corresponds to type-I superconductors with $\kappa \ll 1$, we find from (9)

$$Q(k) = \frac{c}{4\pi}\lambda_L^{-2}\left\{1 + \frac{\pi k\xi(T)}{4\sqrt{2}}\sqrt{\frac{Gi}{t}}\left(1 - \frac{4\sqrt{2}}{\pi k\xi(T)}\right)\right\}. \tag{12}$$

For the penetration depth determined from the formula $\lambda = \int_0^\infty B(x)dx/H$, we find the following result from (12) for the case of specular reflection at the boundary:

$$\lambda^{-2} = \lambda_L^{-2} \left[1 + \frac{1}{2\sqrt{2}\kappa}\sqrt{\frac{Gi}{t}}(1 - 2\sqrt{2}\kappa)\right], \quad \kappa \ll 1. \tag{13}$$

Here the nonlocal nature of the diamagnetic response is important for even the first fluctuational correction to the penetration depth.

The range of applicability of the results derived above is determined by the applicability of the Ginzburg–Landau theory, the assumption that the fluctuations in the order parameter are small, and the validity of ignoring the fluctuations of the magnetic field. These conditions impose the requirements

$$k \ll \xi_0^{-1}; \quad \max\{Gi, \ \kappa^{-6}Gi\} \ll |t| \ll 1. \tag{14}$$

The condition $|t| \gg Gi/\kappa^6$ is important for a type-I superconductor with $\kappa < 1$. Because of this condition, in describing small fluctuations in the order parameter one can ignore the circumstance that the fluctuations in the magnetic field make the superconducting transition a first-order phase transition.[7] In particular, we find $(Gi/|t|)^{1/2}\kappa^{-1} \ll \kappa^2$ from this condition. Consequently, the second term in (13) is always small in comparison with the first, as it should be.

These results apply to the case of a bulk (three-dimensional) superconductor. Corresponding questions for samples of reduced dimensionality and also the spatial dispersion of the fluctuational conductivity will be discussed separately.

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