Anyons in the magnetic field and fractional quantum Hall effect

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The incompressibility of a system of free anyons in the external magnetic field at special filling fractions is discussed. A new sequence of fractional quantum Hall effect states of the first level of hierarchy is proposed.

The characterization of all the possible incompressible states is one of the main goals of a complete theory of the fractional quantum Hall effect (FQHE). In addition to the main states with the filling factors $\nu = 1/m$ (m is an odd integer) described by the Laughlin's variational wave function,[1] a large number of other rational filling fractions have been observed experimentally. At present, their classification is far from being completed. The first construction of the hierarchy of filling fractions[2,3] was based on the quasiparticle wave functions of the Laughlin's type (or, equivalently, on the idea of condensation of the effective bosonic particles). The notion of binding of the $m-1$ flux quanta by electrons and the integer quantum Hall effect of these composite objects was proposed by Jain.[4] The corresponding states are the lowest level of the hierarchy of FQHE states. The field-theoretical realization of this scenario was proposed in Refs. 5 and 6. The Coulomb interaction is essential in the formation of these states. Recently, the existence of a wider class of FQHE states of the first level of hierarchy was argued by Ma and Zhang.[7] It is known that the quasiparticles (quasiholes) in the basic $1/m$ FQHE states are fractionally charged and that they obey the fractional statistics. The latter can be understood either by using the variational wave functions.[3,8] or in the framework of the description in terms of the bosonic variables,[9] where the quasiparticles correspond to the vortices which acquire the nonzero energy due to the Coulomb interaction. In Ref. 7 the incompressibility of the system of anyons in the magnetic field at special filling factors was claimed. If an iteger number of the Landau levels in the effective magnetic field is filled, the system is incompressible even in the absence of the interaction, which is important in this scenario only for the formation of the quasiparticles. The arguments[7] based on the perturbation theory in the small deviation from the Fermi statistics are not too convincing. Actually, this deviation is not small. Moreover, the formation of the mean field cannot be described on the basis of the perturbation theory. The system of free anyons obeying the $1/N$ statistics, for example, is compressible even for a large N.

In the present letter we propose the arguments in favor of the incompressibility of the system of anyons at special filling fractions, using the description in terms of the Chern–Simons theory. We obtain a wider class of the filling fractions of FQHE states of the first level of hierarchy, which differs from that of Ref. 7, because of the power of the singular gauge transformation of the fermionic wave function into another antisymmetric (fermionic) wave function. Our filling fractions are different from those found by Jain.[4] The analysis of quasiparticle and quasihole states has been performed.

Let us consider the system of anyons (quasiparticles) with the statistical parameter α with respect to Bose statistics in the external magnetic field H_q (the charge of the anyons equals 1). The filling factor for anyons $\nu_a = 2\pi nq/H_q$, where n_q is the density of anyons. Let us transform the multiparticle (Bose) wave function Φ according to ($z_i = x_i + iy_i$, $i,j = 1,\dots N$)

$$\Phi(\vec{x}_1,\dots\vec{x}_N) = \prod_{i<j}\left(\frac{z_i - z_j}{|z_i - z_j|}\right)^p \Psi(\vec{x}_1,\dots\vec{x}_N), \tag{1}$$

where p is an odd integer. The new wave function Ψ corresponds to Fermi statistics. In the second-quantized form, introducing the gauge field a_μ, we obtain the Lagrangian

$$L = i\psi^+\partial_0\psi + a_0(\psi^\pm\psi - n_q) + \frac{1}{2M}\psi^+(\vec{\partial} - i\vec{a} - i\vec{A}_1)^2\psi - \frac{1}{4\pi(\alpha - p)}\epsilon^{\mu\nu\alpha}a_\mu\partial_\nu a_\alpha, \tag{2}$$

where the field ψ obeys Fermi statistics, $\vec{A}$ is the vector potential corresponding to the uniform magnetic field of magnitude $H_{\rm eff} = H_q + 2\pi n_q(\alpha - p)$, and M is the anyon mass. The integer number of Landau levels in the effective magnetic field is filled at $\nu_a = (p + 1/n - \alpha)^{-1}$, where n is an integer, $n \neq 0$ (the sign of $H_{\rm eff}$ is not fixed). The fermion propagator has a gap equal to the energy difference between two Landau levels, and the fermions can be integrated out in the action (2). In the one-loop effective action for the field a_μ, there is no cancellation of the Chern–Simons term. Moreover, the non-renormalization theorem[10] (for its application to the system of anyons see Refs. 6 and 11) leads to the conclusion that there are no perturbation-theory corrections to the Chern–Simons term in the effective action.[12] This means that the collective excitation corresponding to the density fluctuations (described by the field a_μ) has a gap (the field a_μ acquires a mass). The single-particle excitations are gapfull due to the filling of the Landau levels. Thus, for example, the imaginary part of the correlator of two electromagnetic currents at small energies is equal to zero: the gapless excitations are absent. Thus, the system at the filling factors considered here is incompressible. It should be noted that the difference between the present case and that of the Chern–Simons theory for the initial $1/m$ state[5,6] is due to the degeneracy of the $\nu = 1/m$ state in the absence of interaction. This degeneracy manifests itself in the form of the vortices (quasiparticles) with zero creation energy.[9] In our case, these quasiclassical solutions are absent due to the non-integer value of $\alpha - p$ in the Langrangian (2), which makes the perturbation theory (2) well defined even without the interaction. Alternatively, the exact wave function of the nondegenerate ground state can be constructed at $\nu_a = (p + 1/n - \alpha)^{-1}$, using the methods of Ref. 13. This ground state corresponds to the classical configuration $a_\mu = 0$ in the effective action for the field a_μ. Thus, the long-wave fluctuations of this field are correctly described by the effective action of the form.[12]

For the FQHE this scenario implies the following filling fractions. For the quasiparticles in the state with a given m we have $H_q = H/m$ and $\alpha = 1/m$. The filling factor for the electrons is $\nu = (1/m)(1 + \nu_a/m)$. Thus we obtain the following quasiparticle states:

$$\nu^{qp} = \frac{1}{m - \frac{n}{np+1}}, \tag{3}$$

where n is an integer ($n \neq 0$) and p is an odd integer such that $n/(np+1) > 0$ ($n > 0$). For the quasiholes we have $H_q = -H/m$ and $\alpha = 1/m$ and

$$\nu^{qh} = \frac{1}{m + \frac{n}{np-1}}, \tag{4}$$

where n is an integer ($n \neq 0$) and p is an odd integer such that $n/(np-1) > 0$. Let us stress once more that Eqs. (3) and (4) represent the FQHE states of the first level of hierarchy. In fact, our construction is in some sense the generalization of Jain's scheme to the first level of hierarchy. The same procedure can be performed both for quasiparticles of the next hierarchy levels and for the quasiparticle excitations of the Jain's states. At $p=1$ the filling factors (3) and (4) coincide with one of the two sequences of filling factors obtained by Jain (and with the sequence of states in Ref. 7). At $n=1$ we obtain the exact formulas of the first hierarcy level: the fermions at the completely filled single Landau level can be transformed into the bosons at zero effective magnetic field, which corresponds to the construction of the hierarchy.[2,3] As we have mentioned, at $p=1$ the states (3) and (4) coincide with the states of Ref. 4 [for $m=3$: (1/3), 2/5, 3/7, 4/9, 5/11,...]. At $p=3$ the $m=3$ sequence reads (1/3), 4/11, 7/19,... for quasiparticles and (1/3), 2/7, 5/17,... for the quasiholes. For $p=5$ the $m=3$ sequences are (1/3), 6/17, 11/31,... and (1/3), 4/13, 9/29,... for the quasiparticles and the quasiholes, respectively. For instance, one can see that the fractions 4/11 and 4/13, which were observed experimentally, are predicted. At the same time, the existence of these states cannot be explained in the framework of the approach of Ref. 4. In general, the interpretation of the observed FQHE states as the states of high hierarchy level of the Haldane–Halperin hierarchical scheme is not quite natural. One can verify that all the FQHE state observed experimentally can be identified either with the states of the lowest level of hierarchy proposed in Ref. 4, or with the states given by Eqs. (3) and (4). As for the filling fractions predicted by both of these two schemes, much work is needed to determine the most adequate one. Clearly, it depends on the dynamics of formation of the quasiparticles, i.e., on their size in comparison with the average distance between the quasiparticles.

[1] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); R. B. Laughlin, Surface Science **141**, 11 (1984).

[3] B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).

[4] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).

[5] A. Lopez and E. Fradkin, Phys. Rev. B **44**, 5246 (1991).

[6] A. A. Ovchinnikov, Pis'ma Zh. Eksp. Teor. Fiz. **54**, 579 (1991) [JETP Lett. **54**, 583 (1991)]; Mod. Phys. Lett. A (to be published).

[7] M. Ma and F. C. Zhang, Phys. Rev. Lett. **66**, 1769 (1991).

[8] D. A. Arovas, R. Schreiffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).

[9] S. M. Girvin, in: *The Quantum Hall Effect.* Eds. R. E. Prange and S. M. Girvin, 1987; C. L. Kane, S. Kivelson, D. H. Lee, and S. C. Zhang, Phys. Rev. B **43**, 3255 (1991); Z. F. Ezawa, M. Hotta, and A. Iwazaki, Tohoku University Preprint TU-376 (1991); Phys. Rev. D **44**, 452 (1991).

[10]S. Coleman and B. Hill, Phys. Lett. B **159**, 184 (1985).
[11]J. D. Lykken, J. Sonnenschein, and N. Weiss, Phys. Rev. D **42**, 2161 (1990).
[12]Y. H. Chen, F. Wilczek, E. Witten, and B. I. Halperin, Int. J. Mod. Phys. B **3**, 1001 (1989); T. Banks and J. D. Lykken, Nucl. Phys. B **336**, 500 (1990); S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. B **340**, 403 (1990).
[13]M. Johnson and G. S. Girvin, Phys. Rev. B **41**, 6870 (1990); S. M. Girvin *et al.* Phys. Rev. Lett. **65**, 1671 (1990); M. Greiter and M. Wilczek, Preprint, 1991.