Effects of an electron–plasmon interaction in a degenerate electron gas

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A theory is derived for plasmorons, i.e., bound states of electrons and plasmons, in a dense, degenerate electron gas.

In a dense, degenerate electron gas, the parameter $g = e^2/\pi\hbar v_F$ is small, and its excitation spectrum can be derived in the approximation of a self-consistent field. The spectrum of electrons is determined from the poles of their Green's function $G(\epsilon,p)$ (Ref. 1):

$$G(\epsilon,p)=\frac{1}{G_0^{-1}(\epsilon,p)-\Sigma(\epsilon,p)};\quad G_0=\frac{1}{\epsilon-\xi},\tag{1}$$

where ξ is the spectrum of an ideal gas, which is linear in the momentum p near the Fermi momentum p_F: $\xi=(p-p_F)v_F$. The plasmon spectrum ω_k has a gap ω_0 and is quadratic in the momentum k at small k. It corresponds to the poles of the screened Coulomb interaction $D(k,\omega)$ (Ref. 1):

$$D(k,\omega)=\frac{4\pi e^2}{k^2}\frac{\omega_0^2}{\omega^2-\omega_k^2}\quad \omega_k^2=\omega_0^2+\frac{3}{5}k^2v_F^2.\tag{2}$$

The very earliest papers on the theory of an electron gas[2–4] revealed that as the electron energy ϵ approaches a certain threshold $\epsilon=\omega_0+\xi$, at which an electron may emit a real plasmon with small k, the contribution of the electron–plasmon interaction to the mass operator Σ in (1) has a strong singularity of a threshold type. Unfortunately, those papers considered only the first order of a perturbation theory in the parameter g; that approach is unacceptable near the threshold, where we have $\epsilon\to\omega_0+\xi$.

Our purpose in the present letter is to derive a rigorous theory of a plasmoron: an electron dressed in a sheath of plasmons. A summation of the entire series of a perturbation theory in the parameter g gives rise to poles in the mass operator Σ (zeros of the function G). These singularities in Σ and G may be interpreted as bound states of electrons and plasmons. This interpretation is suggested by the analogy with superconductivity theory, in which Cooper pairing of electrons also gives rise to poles in Σ, i.e., zeros in G.

We first determine Σ in the first approximation in the parameter g:

$$\Sigma_0(\epsilon,\vec{p})=i\int G_0(\vec{p}+\vec{k},\epsilon+\omega)D(\vec{k},\omega)\frac{d^3k\,d\omega}{(2\pi)^4}.\tag{3}$$

Using (1) and (2), we find from (3), after an integration over ω,

$$\Sigma_0(\epsilon,\vec{p})=\Sigma_0^+(\epsilon,\vec{p})+\Sigma_0^-(\epsilon,\vec{p})\ ,$$

$$\Sigma_0^+=\frac{e^2\omega_0}{(2\pi)^2}\int\frac{d^3k}{k^2}\frac{(1-n_{\vec{p}+\vec{k}})}{\epsilon-\omega_k-\xi-\vec{k}\vec{v}_F+i\delta}\ ,\tag{4}$$

$$\Sigma_0^-=\frac{e^2\omega_0}{(2\pi)^2}\int\frac{d^3k}{k^2}\frac{n_{\vec{p}+\vec{k}}}{\epsilon+\omega_k-\xi-\vec{k}\vec{v}_F-i\delta},$$

where n_p is the Fermi momentum distribution of the electrons. The discussion can be restricted to one of the quantities Σ_0^+,Σ_0^-, since these two quantities are related at small values of ξ by

$$\Sigma_0^-(\epsilon,\xi)=-\Sigma_0^+(-\epsilon,-\xi),\quad \xi\equiv(p-p_F)v_F.\tag{5}$$

Here are the limiting expressions for the real and imaginary parts of Σ^+, near the threshold value $\epsilon\approx\omega_0$:

$$\mathrm{Re}\Sigma_0^+ = -\omega_0 \frac{g}{8} \ln^2 \frac{\omega_0}{|\epsilon - \omega_0|}, \qquad \text{for} \quad \xi = 0,$$

$$\mathrm{Re}\Sigma_0^+ = -\omega_0 \frac{g}{4} \ln^2 \frac{\omega_0}{|\xi|}, \qquad \text{for} \quad \epsilon = \omega_0, \tag{6}$$

$$\mathrm{Im}\Sigma_0^+ = -\omega_0 g \frac{\pi}{2} \ln \frac{\xi\omega_0}{|\epsilon - \omega_0 - \xi|^2}, \qquad \text{for} \quad \xi > 0 \text{ and } \epsilon \to \omega_0 + \xi.$$

We see that, even in first order in g, the mass operator Σ does indeed acquire strong singularities of a threshold type, both for particles, under the condition $\epsilon = \omega_0 + \xi$ ($\xi > 0$), and for holes, under the condition $\epsilon = -\omega_0 + \xi$ ($\xi < 0$). The following terms of the series of a perturbation theory in g can be incorporated by the methods developed in Refs. 5 and 6, by summing the sequence of diagrams which exist in the threshold approximation. Since the parameter g is small, we can ignore the other diagrams, which are not of a threshold nature. In particular, there is no need to refine the "seed" Green's function G_0 in (3). If $\epsilon \approx \omega_0$, the result of this summation is

$$\Sigma^+ = \frac{\Sigma_0^+}{1 + \Sigma_0^+/\omega_0}. \tag{7a}$$

If in contrast we have $\epsilon \approx -\omega_0$, then

$$\Sigma^- = \frac{\Sigma_0^-}{1 - \Sigma_0^-/\omega_0}. \tag{7b}$$

Comparing (7a) and (7b), we find a compact expression for Σ which is valid for both $\epsilon \approx \omega_0$ and $\epsilon \approx -\omega_0$:

$$\Sigma = \frac{\Sigma_0}{1 + G_0 \Sigma_0}. \tag{7}$$

This result corresponds to a simple expression for the Green's function G:

$$G = G_0(1 + \Sigma_0 G_0). \tag{8}$$

This expression is valid far from singularities of the seed function G_0; it is exact near the threshold. Expression (8) can also be derived in a different way, by developing a threshold diagram technique for G, rather than for Σ. Direct calculations show that a perturbation theory in the parameter g is applicable for G. For the same reason, the amplitude for the scattering of an electron by a plasma does not have any strong singularities of the threshold type.

Incorporating the electron–plasmon interaction has thus given rise to new singularities in the Green's function G [see (6) and (8)]. At $\epsilon = \omega_0 + \xi$ ($\xi > 0$) and $\epsilon = -\omega_0 + \xi$ ($\xi < 0$), the function G has the logarithmic poles

$$G \propto ig/\omega_0 \ln /\epsilon - \omega_0 - \xi/, \quad \text{for} \quad \epsilon \to \omega_0 + \xi \qquad G \propto ig/\omega_0 \ln /\epsilon + \omega_0 - \xi/, \quad \text{for} \quad \epsilon \to -\omega_0 + \xi. \tag{9}$$

At energies $-\omega_0 < \epsilon < \omega_0$ the function G has two zeros. One corresponds to a bound state of a particle and a plasmon, and the other to a bound state of a hole and a plasmon. The binding energy (Δ) of these states is exponentially small; at $\xi = 0$ ($p = p_F$) we have

$$\Delta \approx \omega_0 \exp\left[-\sqrt{\frac{8}{g}}\right]. \tag{10}$$

The bound states exist in a narrow momentum interval Δp near the Fermi momentum p_F ($\Delta p = p - p_F$):

$$\Delta p \approx \frac{\omega_0}{v_F} \exp\left[-\frac{2}{\sqrt{g}}\right]. \tag{11}$$

The logarithmic singularities of the function G make a contribution to the one-electron density of states which is seen in the soft x-ray spectra of a metal as plasma satellites shifted by the plasmon frequency ω_0 from the fundamental band of the spectrum. The same singularities contribute to the momentum distribution of the electrons, which can be measured in experiments on the scattering of hard x rays in metals.

To single out these contributions, we recall the relationships between (on the one hand) the one-electron density of states $\rho(\epsilon)$ and the electron momentum distribution n_p and (on the other) the imaginary part of the one-electron Green's function G:

$$\rho(\epsilon) = \frac{1}{\pi} \int |\mathrm{Im} G(p,\epsilon)| d^3p, \tag{12}$$

$$n_p = \frac{1}{\pi} \int_{-\infty}^{0} \mathrm{Im} G(p,\epsilon) d\epsilon.$$

Using (4) and (8), we find from (12) expressions for $\rho(\epsilon)$ which are applicable near the threshold values of ϵ ($|\epsilon| - \omega_0 \ll 1$):

$$\rho(\epsilon) = \rho(\omega_0) + \frac{4\pi p_F^2}{v_F} g \left(\frac{10}{3}\right)^{1/2} \left(\frac{\epsilon}{\omega_0} - 1\right)^{1/2} \tag{13}$$

for $\epsilon > \omega_0$ and

$$\rho(\epsilon) = \rho(-\omega_0) + \frac{4\pi p_F^2}{v_F} g \left(\frac{10}{3}\right)^{1/2} \left(-\frac{\epsilon}{\omega_0} - 1\right)^{1/2}$$

for $\epsilon < -\omega_0$.

For $-\omega_0 < \epsilon < \omega_0$ these effects do not influence the value of $\rho(\epsilon)$. It can be seen from (13) that the electron–plasmon interaction makes a nonanalytic contribution to the one-electron density of states $\rho(\epsilon)$. The derivative of ρ with respect to ϵ diverges in a square-root fashion at $\epsilon = \pm\omega_0$.

A corresponding nonanalytic behavior is seen in the dependence of the momentum distribution n_p on p. The derivative of the electron–plasmon contribution, Δn_p, with respect to p diverges logarithmically at the Fermi boundary:

$$\frac{\partial \Delta n_p}{\partial p} = \frac{g}{2}\frac{v_F}{\omega_0} \ln \left| \frac{p}{p_F} - 1 \right|. \tag{14}$$

It would be interesting to see an experimental confirmation of the ϵ dependence of ρ in (13) and of the p dependence of n_p in (14).

[1]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, *Quantum Field-Theoretical Methods in Statistical Physics*, Pergamon, New York, 1965.
[2]R. A. Ferrell, Phys. Rev. **112**, 812 (1958).
[3]L. Hedin, B. I. Lundgvist, and S. Lundgvist, Solid State Commun. **5**, 237 (1967).
[4]L. Hedin and S. Lundgvist, Solid State Phys. **23**, 1 (1969).
[5]L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **36**, 1168 (1959) [Sov. Phys. JETP **9**, 830 (1959)].
[6]I. B. Levinson and É. I. Rashba, Usp. Fiz. Nauk **111**, 683 (1973) [Sov. Phys. Usp. **16**, 892 (1973)].

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