Convective heat transfer in layered high-T_c superconductors

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The convective heat flux in a planar or layered high-T_c superconductor is not determined exclusively by the ordinary transfer of thermal energy by normal excitations. It is also determined by a flux of macroscopic excitations of a vortex type, which carry the excess entropy of electrons localized in the vortex cores.

Highly anisotropic single crystals of high-T_c superconductors, in which a crossover from a 3D behavior to a 2D behavior occurs in a fluctuation region,[1] exhibit nonlinear current-voltage characteristics $V \propto I^a(T)$, where $a(T) = 1 + q^2/2kT$, $q^2 = \Phi_0^2/(8\pi^2\lambda_{\text{eff}})$, Φ_0 is the flux quantum, and λ_{eff} is the effective magnetic-field

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penetration depth.[2–5] This resistive behavior of single crystals is a manifestation of a vortex phase transition [a (Berezinskiĭ–) Kosterlitz–Thouless transition] in the superconducting layers of the superconductors.

Recognizing that macroscopic excitations in the form of magnetic vortices of various polarities exist in the Cu–O layers in the highly anisotropic high-T_c superconductors, we should note that 2D vortex excitations of this sort are capable of transporting an entropy S_φ (Ref. 6). The reason for this effect is that the entropy density in the core of a vortex is higher than that in the surrounding superconducting phase. One would expect that heat-transfer effects in high-T_c superconductors in a zero magnetic field would have characteristics different from those of conventional superconductors. In the present letter we show that the "fluctuational" part of the convective heat transfer, Δk, in high-T_c superconducting systems is substantial near the temperature of a vortex phase transition in the layered system, T_{2D}. In the calculations we assume that the temperature profile in the sample is nonuniform only along the Cu–O planes. We also assume that there is no vortex pinning.

If the temperature is spatially nonuniform, thermally induced vortex fluctuations (regardless of their polarity) should move along the direction of the heat flux vector in the temperature profile, at a velocity[6]

$$\vec{v}_L = -S_\varphi \eta^{-1} \vec{\nabla} T \tag{1}$$

(η is the viscosity), thereby performing a transfer of heat.

We take the flux density of the planar vortex flow to be $J_f = n v_L$, where n is the density of vortex molecules and of free vortices of various polarities. We assume that in the course of its motion a vortex convects an amount of heat TS_φ. We can then write the following expressions for the components of the heat flux density along a Cu–O plane:

$$\vec{q}_i = -k_{ii} \vec{\nabla} T + T S_\varphi n \vec{v}_L \qquad i = 1, 2, \tag{2}$$

where k_{ii} includes the electronic, phonon, and convective components of the heat transfer which are characteristic of conventional 3D superconductors.

In writing expression (2) for the heat transfer in an anisotropic superconductor, we made use of the fact that the crystals of high-T_c superconductors have a symmetry axis and a symmetry plane. We have directed the z axis along the symmetry axis of the crystal, i.e., the c axis. In this case the array of heat-transfer coefficients k_{ij}, which are the components of a second-rank tensor, is diagonal.[7]

Since the flow of 2D vortices occurs in all planes, we can take S_φ in (2) to mean the entropy of an ordinary 3D vortex per unit length of the vortex.

Substituting (1) into (2), we find the following result for layered high-T_c superconductors:

$$\tilde{k}_{ii} = k_{ii} + T S_\varphi^2 \eta^{-1} n_L. \tag{3}$$

At low temperatures ($T < T_{2D}$), at which only vortex molecules can be nucleated in the system, the density of these molecules is

$$n_b \cong \frac{2\Pi}{\xi^4} \int_{\xi}^{\infty} dr\, r \exp(-E(r)/T). \quad (4)$$

The energy required to form a pair of vortices is

$$E(r) = 2(H_c^2/8\pi)\pi\xi^2 d + q^2 \ln(r/\xi) = q^2(1/4 + \ln(r/\xi)), \quad (5)$$

where ξ is the coherence length in the plane of the layer, and d is the thickness of a superconducting monolayer. Above 2D, a vortex plasma with a density

$$n_f = c\xi_+^{-2} \quad (6)$$

appears against the background of the molecules which have not decayed, with a maximum size ξ_+. Here, according to Kosterlitz,[8] we have

$$\xi_+(T) = a\xi(T) \exp[b(T_{c0} - T_{2D})/(T - T_{2D}]^{1/2}, \quad (7)$$

where T_{c0} is the mean-field superconducting transition temperature. The density of vortex molecules above T_{2D} is determined by (4), where we should use ξ_+ as the upper limit. The total number of vortices is then

$$n = \frac{2\pi}{\xi^2}\frac{\exp(-q^2/4T)}{q^2/T - 2}\left[1 - \left(\frac{\xi_+}{\xi}\right)^{2-q^2/T} \vartheta(T - T_{2D})\right] + \frac{c\vartheta(T - T_{2D})}{\xi_+^2}. \quad (8)$$

Figure 1 shows the temperature dependence $n(T)$ in units of ξ^{-2}. The coefficients a, b, and c in (7) and (8) are on the order of unity.

To calculate $\Delta k = \tilde{k}_{ii} - k_{ii}$, we note that the entropy of a vortex can be described well over the entire temperature range from absolute zero up to T_{c0} by the approximate expression[9] $S_\varphi(T) = S_0(1 - (T/T_{c0})^p)T/T_{c0}$, where $p = 2$–3, and $S_0 \sim 10^{-8}$ erg/(cm·K) (calorimetric studies of high-T_c samples[10] yield values for S_φ which are

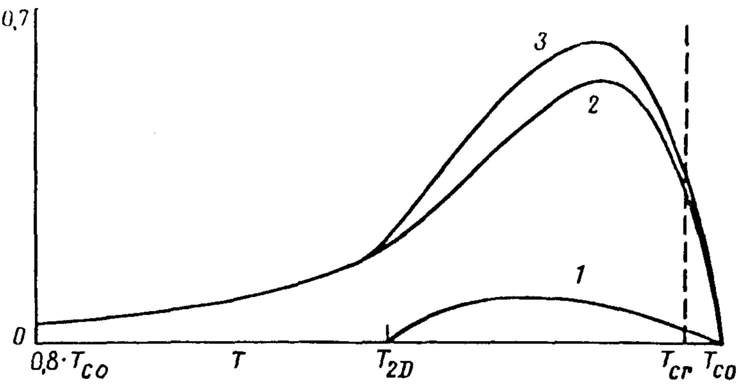

FIG. 1. 1—Density of free vortices, n_f; 2—density of vortices bound in pairs; 3—total density n. These concepts are meaningful at $T < T_{cr}$, where T_{cr} is the crossover temperature.

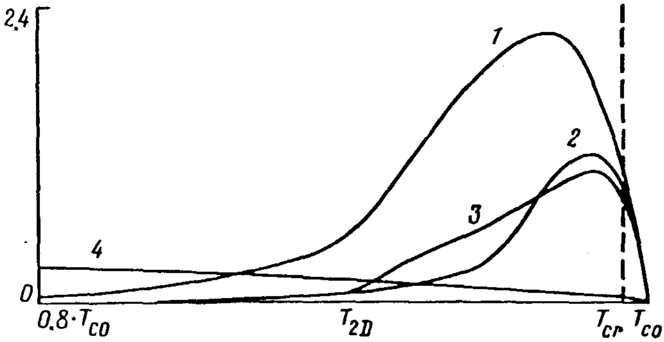

FIG. 2. Heat-transfer correction Δk as a function of the temperature for several parameter values. 1—$\tau_c = 0.1$, $\epsilon_c = 1$; 2—$\tau_c = 0.05$, $\epsilon_c = 1$; 3—$\tau_c = 0.1$, $\epsilon_c = 2$. Line 4 is the entropy of a vortex, $S_\varphi(t)/S_0$.

an order of magnitude lower than those in conventional systems). Estimating η from the familiar relation $\eta = \Phi_0^2/(c^2\xi^2\rho_n)$ (ρ_n is the normal electrical resistivity), we can then write Δk as $\Delta k = Af(t)$, where $A = S_0^2 T_{c0}\rho_n e^2/h^2\pi^2$, and the function f, a function of the dimensionless temperature $t = T/T_{c0}$, is shown in Fig. 2 for various values of the constant $\tau_c = (T_{c0} - T_{2D})/T_{c0}$ and of the "dielectric constant" ϵ_c of the vortex plasma (these are properties of the material).[11]

Consequently, in planar superconductors or in layered high-T_c superconducting compounds, in which a vortex phase transition occurs, heat is transferred not exclusively by the scattering of quasiparticles by each other but also by a thermal induction of vortex fluctuations and a transfer of thermal energy by these fluctuations along the direction of their velocity. This effect is analogous to the Ettinghausen effect[6] for a conventional superconductor in a dynamic mixed state. In that case, a vortex flux flow or flux tube is nucleated at one edge of the sample (where heat is absorbed) and annihilates at the other edge (where heat is evolved).

The convective part of the heat transfer of a high-T_c superconductor, which is determined by the normal component of the current, is two orders of magnitude higher than the convective heat transfer in conventional superconductors according to the estimates of Ref. 12. Ginzburg showed[12] that the heat transfer which is measured should have a maximum because of a large normal convective component. This maximum is observed on plots of the heat transfer for both ceramic samples and single crystals.[13] When the fluctuational increment Δk is taken into account, this maximum should be of larger scale.

Let us estimate Δk. The function $f(T)$ at the maximum can be estimated from $f_{\max} \cong 2\pi p\tau_c/e \cong \tau_c$. Comparing $\Delta k_{\max}$ with the electron thermal conductivity [here we use the Wiedemann–Franz law $k_{ee} = (1/3)\pi k_b/e)^2 T\sigma_n$], we find

$$\frac{\Delta k_{max}}{k_{ee}} \cong \frac{6p}{\pi^3 e}\left(\frac{S_0\rho_n}{k_b}\right)^2 \frac{1}{R_0^2}\tau_c, \tag{9}$$

where the characteristic resistance is $R_0 = h/e^2 = 4.1k\Omega$. The value $\rho_n \simeq 100\ \mu\Omega$, which is representative of the high-T_c superconductors,[14] then leads us to $\Delta k_{\max}/k_{ee} \cong (1\text{–}10)\tau_c$. In the highly anisotropic Bi- and Tl-based high-T_c superconductors, the latter parameter reaches values of[5] 4×10^{-2}, so values $\Delta k_{\max}/k_{ee} \simeq 10^{-1}$ are possible.

In the picture drawn above, the anisotropy of the compounds is important only to the extent that it leads to the satisfaction of the inequality $\tau_f \gg \tau_{cr}$, which we discussed in the first sentence of this paper. Under this condition, the transition temperature T_{2D} itself varies over the interval $T_{KT} < T_{2D} < T_{c0}$, depending on the degree of anisotropy. This is an exceedingly narrow interval (T_{KT} is the temperature of the 2D transition in an isolated monolayer). The parameter τ_c also depends weakly on the degree of anisotropy. Specifically, we have[1]

$$\tau_c = \tau_{KT}\left[1 - g/\ln^2\left(\frac{M}{m}\frac{d^2}{\xi_{\parallel}^2}\tau_{KT}\right)\right], \tag{10}$$

where $g \simeq 1$, and M and m are the effective masses of the carriers along the c axis and in the ab plane, respectively.

A possible pinning of 2D vortices could hardly have any important effect on the vortex flow near T_{KT}, since we have[11] $U_p \simeq (H_c^2/8\pi)\pi\xi^2 d = q^2(T_{KT})/8 = T_{KT}/2$.

We note in conclusion that for the longitudinal flux flow of vortices in a nonuniform temperature field, an additional amount of heat $\Delta Q(T,r) = -d/dT(nv_L TS_\varphi/\partial T)$, which is proportional to $(\vec{\nabla} T)^2$, will be evolved in the superconductor. This term can be omitted from the expression for the dissipation function in the heat balance equation.

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