Inversion wake of a moving Abrikosov vortex in a magnetic superconductor

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The magnetic structure of a moving Abrikosov vortex in a magnetic superconductor is analyzed. The magnetic subsystem seriously distorts the shape of the vortex, leading to a slow, power-law decay of the vortex field at large distances. A moving vortex should leave an "inversion wake."

A large number of magnetic superconductors are now recognized. High-T_c compounds of the types REBaCuO, RECuO, etc., where RE is a rare-earth ion, have now joined the ternary compounds[1] on the list of materials exhibiting a coexistence of a superconductivity and a magnetism. Finally, a strong antiferromagnetic correlation of the copper spins in CuO_2 planes in the superconducting states is one of the most important features of the high-T_c materials. (An extensive bibliography on this topic is given in Ref. 2.)

In type-II magnetic superconductors, the magnetic field penetrates in the form of Abrikosov vortices[3] and induces a magnetization of the magnetic subsystem over a distance on the order of the London depth λ around the normal core of the vortex. A sufficiently high transport current flowing through the superconductor in the mixed state sets the system of Abrikosov vortices in motion.[4] In this letter we report a study of the magnetic field structure of a slowly moving, isolated Abrikosov vortex. We show that the magnetic subsystem substantially distorts the shape of the vortex. We predict that a moving vortex will leave an "inversion wake."

We assume an electrodynamic coupling of an undamped current with the magnetic subsystem. As usual in the London approximation, we ignore the structure of the core of the vortex in accordance with the condition $\lambda \gg \zeta$, where ζ is the correlation length. Finally, in treating the case of isolated vortices, we actually assume that the distance between vortices satisfies the $d \gg \lambda$ [but see the discussion of expression (10)].

Following Ref. 5, we work from Maxwell's equations for the magnetic induction $\vec{b}(\vec{r}) = \vec{\nabla} \times \vec{a}(\vec{r})$ [$\vec{a}(\vec{r})$ is the vector potential], which is determined by the sum of the magnetic field $\vec{h}(\vec{r})$, generated by the undamped current $\vec{j}(\vec{r})$, and the magnetization $\vec{m}(\vec{r})$. This induction satisfies the equation (c is the velocity of light)

$$\vec{\nabla} \times \vec{b}(\vec{r}) = \frac{4\pi}{c}\vec{j}(\vec{r}) + 4\pi \vec{\nabla} \times \vec{m}(\vec{r}). \tag{1}$$

In the London potential gauge [$\vec{\nabla}\vec{a}(\vec{r}) = 0$], the current, the potential, and the phase of the order parameter, $\psi(\vec{r})$, are related by[6]

$$\vec{j}(\vec{r}) = -\frac{c}{4\pi\lambda^2}\left(\vec{a}(\vec{r}) + \frac{\Phi_0}{2\pi}\vec{\nabla}\Psi(\vec{r})\right), \tag{2}$$

where Φ_0 is the flux quantum. The phase of the order parameter satisfies the condition

$$\vec{\nabla}\times\vec{\nabla}\Psi(\vec{r}) = -2\pi\vec{l}\delta(\vec{r}-\vec{r}_0), \tag{3}$$

where $\vec{l}$ is the unit vector of a vortex at the point $\vec{r}_0$ and $\delta(\vec{r})$ is the Dirac δ-function. Using expression (2) for the current, and (3) for the source, we rewrite Eq. (1) as

$$\lambda^2 \text{curlcurl}\,\vec{h}(\vec{r},t) + \vec{b}(\vec{r},t) = \Phi_0\vec{l}\delta(\vec{r}-\vec{v}t), \tag{4}$$

where we have assumed $\vec{r}_0 = \vec{v}t$ for a vortex in uniform motion ($\vec{v}$ is the velocity of the vortex). Another way to derive Eq. (4) is demonstrated in Ref. 7 (among other places).

A two-dimensional Fourier transformation in the plane perpendicular to the vortex reduces the equation for the magnetic field to the algebraic equation

$$\vec{h}_{\vec{k}}(t) + 4\pi\hat{\chi}\vec{h}_{\vec{k}}(t) - \lambda^2\vec{k}\times(\vec{k}\times\vec{h}_{\vec{k}}(t)) = \Phi_0\vec{l}\exp[-it(\vec{k}\vec{v})]. \tag{5}$$

Here we have introduced a magnetic susceptibility $\vec{m}_{\vec{k}} = \hat{\chi}\vec{h}_{\vec{k}}$ in the standard fashion. We see that the field has a time (t) dependence $\vec{h}_{\vec{k}} \sim \exp[-it(\vec{k}\vec{v})]$. Assuming that the axis of the vortex is oriented along one of the principal axes of the tensor $\hat{\chi}$, and allowing for the effect of the operator $\hat{\chi}(\omega)$ on the function $\exp(-i\omega t)$, we find a solution of Eqs. (5) for the Fourier components:

$$\vec{h}_{\vec{k}}(t) = \vec{l}\Phi_0\frac{e^{-it(\vec{k}\vec{v})}}{1 + 4\pi\chi_{ll}(\vec{k},\vec{k}\vec{v}) + \lambda^2k^2}. \tag{6}$$

Since the relation $\lambda \gg a$ holds (a is a lattice constant of the crystal), it is natural to use a hydrodynamic description of the magnetic subsystem. We restrict the discussion to the paramagnetic temperature region. In this case we have the following expression for the susceptibility:[8]

$$\chi(\vec{k},\omega) = \chi'(\vec{k},\omega) + i\chi''(\vec{k},\omega) = i\frac{\chi_0 Dk^2}{\omega + iDk^2}. \tag{7}$$

Here χ_0 is the static magnetic susceptibility, and the spin diffusion coefficient for two-dimensional Heisenberg magnetic materials is[9] $D = (1/3)\ (2\pi)^{1/2}\ Ja^2[S(S+1)]^{1/2}$ (J is the intralayer-exchange parameter, and S is the spin). Strictly speaking, superconducting currents screen the long-wave part of the exchange interaction and of the electromagnetic interaction, renormalizing the parameters of the magnetic system.[10] However, since we are interested below in only order-of-magnitude estimates, in dealing with the paramagnetic temperature region we will ignore this point.

Substituting (7) into (6), and carrying out some straightforward manipulations, we find

$$\vec{h}_{\vec{k}}(t) = \vec{l}\Phi_0\frac{q - iv\cos\varphi/v_0}{(q-iq_1)(q+iq_2)(q-iq_3)}\exp\left(-i\frac{t}{\lambda}vq\cos\varphi\right). \tag{8}$$

Here $v_0 \equiv D/\lambda, q \equiv \lambda k, (k,\varphi)$ are the polar coordinates of the wave vector $\vec{k}$ in the xy basal plane, and the parameters $q_{1,2,3}$ determine the effective field penetration depth, in which the magnetism and the anisotropy due to the motion of the vortex are taken into account. We are directing the velocity along the x axis. Using (8), we find an integral representation for the magnetic field distribution, but it is a fairly difficult matter to find the explicit functional dependence $h(\vec{r},t)$.

Values $\chi_0 \sim 10^{-3}$–10^{-5} are typical of antiferromagnets. We assume that the motion of the vortex is slow, $v \ll v_0$. The characteristic velocity $v_0 \sim SJa(a/\lambda)$ is smaller by a factor $a/\lambda \sim 10^{-2}$–10^{-3} than the spin-wave velocity $v_s \sim JSa$. For the CuO_2 layers, the pronounced intralayer exchange makes the spin-wave velocity fairly high: $v_s \sim (0.5$–$1.3) \times 10^7$ cm/s (Ref. 11), i.e., $v_0 \sim 10^4$–10^5 cm/s. The highest vortex velocities which have been observed experimentally to date are considerably lower:[12] $V_v \approx 6.6 \times 10^3$ cm/s.

We can now work by perturbation theory to find an explicit expression for the parameters $q_{1,2,3}$. Within terms of order $(v/v_0)^2$ and $(4\pi\chi_0)^2$ we find

$$q_{1,2} = 1 + 2\pi\chi_0 \pm 2\pi\chi_0 \frac{v}{v_0} \cos\varphi,$$

$$q_3 = (1 - 4\pi\chi_0) \frac{v}{v_0} \cos\varphi.$$

In the same approximation, the field distribution is described by the integral

$$h(\vec{r},t) = \frac{\Phi_0}{\lambda^2(2\pi)^2} \int d^2q \exp[i\rho q \cos(\varphi - \theta)]$$
$$\times \left\{ 1 - 4\pi\chi_0 \frac{v}{v_0} \cos\varphi (q - iq_3)^{-1} \right\} (q - iq_1)^{-1}(q + iq_2)^{-1}, \tag{9}$$

where $\lambda^2\rho^2 = (r\cos\varphi_0 - vt)^2 + (r\sin\varphi_0)^2$, $\cos\theta = (r\cos\varphi_0 - vt)/\lambda\rho$, and (r,φ_0) are the polar coordinates of the radius vector $\vec{r}$.

Let us look at the results of an integration of (9) for the "near" region of a vortex, which is the region of greatest interest: $\zeta \ll \rho \ll \lambda v_0/v$. Assuming $v/v_0 \sim 10^{-2}$–10^{-3}, we see that this region is a few tens of λ in size. We have

$$h(\vec{r},t) = \frac{\Phi_0}{2\pi\lambda^2} \left\{ K_0(\rho/\lambda_m) + 2\pi\chi_0 \frac{v}{v_0} \cos\theta \left[\lambda/\rho - \frac{1}{2} K_1(\rho/\lambda) - \frac{\pi}{2}\frac{\rho}{\lambda} IL_0(\rho/\lambda) + \frac{\pi}{2} IL_1(\rho/\lambda) \right] \right\}. \tag{10}$$

Here we have introduced $IL_\nu(x) \equiv I_\nu(x) - L_\nu(x)$, and $K_\nu(x)$, $I_\nu(x)$, and $L_\nu(x)$ are modified Bessel and Struve functions, respectively. Here $\lambda_m^{-1} = \lambda^{-1}(1 + 2\pi\chi_0)$. The first term in braces (curly brackets) in (9) was integrated with an accuracy to terms $[2\pi\chi_0(v/v_0)]^2$. The first corrections to the integral of the second term are on the order of $4\pi\chi_0(v/v_0)^2$. It is because we are discarding these terms that we had to impose the limitation on ρ above.

Here are the basic features of the magnetic field distribution of a moving Abrikosov vortex according to expression (10):

1) At $v=0$, the field is the same as the standard field, within the "magnetic" renormalization of the penetration depth.

2) With $\chi_0=0$, the field distribution is the same as the standard distribution[3] within a Galilean transformation of the coordinate system.

3) In general, the field is asymmetric under reflection in the plane perpendicular to the direction of motion, and in the region $\zeta \ll \rho \ll \lambda v_0/v$ the field falls off slowly, by a power law, with distance.

4) Because of the angular factor, a moving vortex is "oblate." The power-law decay at $\theta \sim 0$, π gives way to an exponential decay at $\theta \approx \pm \pi/2$.

5) An extended "wake" is left behind the moving vortex in the basal plane. Inside this wake the relation $h<0$ holds; i.e., the magnetic field is directed opposite the total magnetic flux in the vortex, $\vec{l}$ (this is thus an "inversion wake").

6) In the region with $h<0$, the field reaches a minimum. A numerical analysis has shown that with the value $\chi_0 \sim 10^{-4}$, which is comparable to the susceptibility of the copper subsystem of the high-T_c superconductors, and with $v/v_0 \sim 10^{-2}$, the field reaches its minimum at a distance $r_9 \sim 10\lambda$, and this minimum value is $H_{\min} = |h(r_0,t)|2\pi\lambda^2/\Phi_0 \sim 10^{-5}$. With $\chi_0 \sim 10^{-2}$ (such values of the magnetic susceptibility are exhibited by the ternary compounds and also by the high-T_c compounds containing rare-earth ions, near the magnetic ordering temperature, $T_N \sim 1\mathrm{K}$), the magnetic field reaches a minimum value $H_{\min} \sim 10^{-3}$.

We know[13,14] that an inversion of the longitudinal component of a magnetic field causes vortices to attract one another. In magnetic superconductors, moving vortices may thus line up in chains. Such an event would have a strong influence on, for example, the operation of Abrikosov-vortex memory devices.

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