Renormalization-group flux on an affine-Virasoro space in connection with sporadic conformal deformations

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An exactly solvable model which embodies a simplified version of the Gerasimov–Lebedev–Morozov program for constructing a complete string theory is proposed and partially analyzed.

The Gerasimov–Lebedev–Morozov (GLM) program which was recently proposed[1,2] for constructing a complete string theory (i.e., off the mass shell) treats string theory as a dynamic theory on a suitable configuration space which includes all $2d$-conformal field theories (*CFT*'s) as "singular points." It also includes all integrable models which lead to a connectedness of the configuration space [i.e., a renormalization-group (RG) interpolation between two *CFT*'s which are fixed points of the RG flux]. In addition, it may contain q-deformations of *CFT*'s and of integrable models, which must be included for completeness of the theory.

1. Our purpose here is to construct a simple model in which the basic ideas of the GLM program permit an explicit constructive realization. We will accordingly discuss those aspects of the GLM program which are reflected in our model. (We will depart slightly from the author's formulation of the GLM program in order to adapt the general positions to a specific model.) (a) The configuration space includes a certain set of *CFT*'s which we denote by $\mathscr{C}$ (i.e., we are not including all the *CFT*'s as in the GLM program). (b) For a certain subset of $\mathscr{C}\times\mathscr{C}$ pairs of *CFT*'s, we construct an RG flux which connects pairs of *CFT*'s. Models based on an RG trajectory are incorporated in the configuration space. (c) We take a field-theory limit in terms of the number of fields [$SU(\infty) - WZNW$]. This step confers the structure of a universal Grassmann manifold on the module space (in target space). It also leads to the appearance of an algebra of double loops. (d) We formulate a dynamic principle on the configuration space of the models. The action is a functional on an effective loop space which arises through a "contraction" of two-dimensional loops involving KM loops. (e) A stochastic quantization of the dynamic system constructed on the configuration space is carried out by the stabilized Greensite–Halpern-Marinari-Parisi procedure.[3,4] (f) Quantum deformations (q-deformations) of the models are carried out. They expand the configuration space.

In this letter we take up the problem of reconstructing the configuration space in accordance with the procedure outlined above on the basis of affine-Virasoro (AV) models[5–8] of a special type.[9] These particular models allow sporadic marginal deformations (which were first observed in Ref. 7) by the space of modules (in the space of targets) which are Grassmann manifolds.

2. We begin by listing the AV models which, in our construction, are analogs of

classical vacuum solutions (CFT's) of complete string theory. As was shown in Ref. 9, solutions of the affine-Virasoro control equation[5,7]

$$t^{ab} = 2kt^{ac}\eta_{cd}t^{db} - t^{cd}t^{ef}f^{a}_{ce}f^{b}_{df} - t^{cd}f^{f}_{ce}f^{(a}_{df}t^{b)e} \tag{1}$$

for the "inertia tensor" t_{ab}, which determines the form of the energy-momentum tensor

$$T(z) = t^{ab}(J_a J_b)(z), \tag{2}$$

constructed on the current algebra $\mathscr{G}^k$,

$$J_a(x)J_b(z) = \frac{k\eta_{ab}}{(x-z)^2} + \frac{if^{c}_{ab}J_c(z)}{(x-z)} + \text{reg}.t, \tag{3}$$

allow multiparameter marginal deformations for certain special selections of the current algebra $\mathscr{G}^k$. We single out two special ansatzes. Specifically, we single out the root ansatz on $SU(N)^1$,

$$T(z) = \sum_{\alpha\in\Phi_+} S_\alpha(E^\alpha E^{-\alpha} + E^{-\alpha}E^\alpha)(z), \tag{4}$$

and a diagonal ansatz on $SO(N)^2$,

$$T(z) = \sum_{i<j}^{N} U_{ij}(X^{ij}X^{ij})(z). \tag{5}$$

With $S_{\lambda_i-\lambda_j} = U_{ij} = -\frac{1}{2}\Sigma_\mu R^\mu_i R^\mu_j$ the module space of marginal deformations of these ansatzes is the Grassmann manifold $G_c(R^{N-1}) = V_c(R^{N-1})/O(c)$, where c is the central charge of the Virasoro algebra, and the Stifel manifold is parametrized by the n-hedral R^μ_i, which satisfies the conditions $\Sigma_i R^\mu_i R^\nu_i = \delta^{\mu\nu}$ and $\Sigma_i R^m_i = 0$. We thus have two classes of non-Abelian AV models at our disposal: $\mathscr{C}^1$ on $SU(N)^1$ and $\mathscr{C}^2$ on $SO(N)^2$.

3. Let us find a generalization of the Knizhnik–Zamolodchikov (KZ) equation for an ansatz of the type in (2). We denote by τ^a the generators of $\mathscr{G}$ in a representation with a senior weight λ and the fields Φ_β which are transformed by this representation. It is not difficult to see that under condition (1) the field $J^\beta_\alpha(z) = t_{ab}[(\tau^a)^\alpha_\beta \times J^b(z) + J^a(z)(\tau^b)^\beta_\alpha]$ is a current, i.e., a prime field of spin 1. Now making use of the constraints on the correlation functions which follow from the existence of a null vector,

$$|\chi> = (L_{-1} - J_{-1})|\Phi>,$$

and also using a Ward identity, we find the following generalization of the KZ equation:

$$\frac{\partial}{\partial z_i} - 2\sum_{j=1}^{n}{}^{(i)}\frac{t_{ab}\tau^a_i \otimes \tau^b_j}{z_i - z_j} < \Phi_1(z_1)...\Phi_n(z_n) >= 0. \tag{6}$$

The anomalous dimensionalities $\Delta(\lambda)$ for the fields Φ_α are found from the solution of the matrix eigenvalue equation

$$\det \| t_{ab}(\tau^a\tau^b)^\alpha_\beta - \Delta(\lambda)\delta^\alpha_\beta \| = 0. \tag{7}$$

4. Let us examine the structure of the Hilbert space of models and find the spectrum of anomalous dimensionalities. In the case of $\mathscr{C}^1$, all functional representations of SU(N) with senior weights $\omega_1,...,\omega_{N-1}$ are integrable. In general, the anomalous dimensionalities are split up completely in all representations, and they depend in a continuous way on the deformation parameters:

$$\Delta_A(\omega_J) = \frac{1}{2}\sum_\mu \Big(\sum_{i\in A} R_i^\mu\Big)\Big(\sum_{j\in A} R_j^\mu\Big), \tag{8}$$

where A is a subset of the indices $A \subset \{1,...,N\}: \# A = J$. The Hilbert space contains N sectors, including the vacuum $|0\rangle$. The sector corresponding to ω_J is generated by J-particle fermion composites $(\psi^+_{i_1}...\psi^+_{i_J})(z)$ with unequal indices and has the dimensionality $\begin{bmatrix} N \\ J \end{bmatrix}$.

In the case of $\mathscr{C}^2$, there are two classes of integrable representations. The first is analogous to that presented above, differing from it in that there are no "conjugate" representations. The second class contains spinor representations with senior weights $\sigma_\pm = \frac{1}{2}(1,1,..., \pm 1)$ for SO$(2n)$ and $\sigma = \frac{1}{2}(1,1,...1)$ for SO$(2n+1)$. A significant property of these representations is that their anomalous dimensionalities are independent of the deformation parameters R^μ_i. Specifically, the generators of SO(N) are, in the spinor representation,

$$t_{ij} = \frac{i}{4}[\gamma_i\gamma_j];$$

in ansatz (5) we thus find $\Delta(\sigma) = c/16$.

We thus see that the $\mathscr{C}^1$ and $\mathscr{C}^2$ AV models are analogs of compactifications of a boson string onto a torus (C^1) and an orbifold (C^2). The spinor fields which have fixed anomalous dimensionalities are analogs of the twist fields of orbifoldized compactifications. In addition, the module space of the AV models, $G_c(R^{N-1})$, is a compact analog of the module space of boson compactifications.[10]

5. We now derive an exact solution of the renormalization-group equations for AV models with sporadic deformations, following the prescription of Ref. 4. For definiteness we consider the class $\mathscr{C}^1$. As the UV fixed point we adopt a point in AVS which lies on the SU$(N)^1$ manifold of AV models with a central charge c. It can be shown that the structure functions S_α in (4) can be represented by the following ansatz on the RG trajectory:

$$S_{\hat{e}_i - \hat{e}_j}(\tau) = R_i^\mu R_j^\nu g_{\mu\nu}(\tau). \tag{9}$$

Here we have the following RG equation for the metric of c planes $g_{\mu\nu}$:

$$\frac{dg_{\mu\nu}(\tau)}{d\tau} = \beta_{\mu\nu}(\{g\}), \tag{10}$$

with the β function

$$\beta_{\mu\nu}(\{g\}) = -g_{\mu\nu} + g_{\mu\kappa} g_{\lambda\nu} \delta^{\kappa\lambda} \tag{11}$$

and the initial conditions

$$g_{\mu\nu}(-\infty) = \delta_{\mu\nu}. \tag{12}$$

The solution of the RG equations is

$$g_{\mu\nu}(\tau) = \delta_{\mu\nu} f_{\mu}(\tau), \tag{13}$$

where

$$f_{\mu}(\tau) = \begin{cases} 1, & \mu \leq c-1 \\ (1+e^{\tau})^{-1}, & \mu = c \end{cases}.$$

The IR fixed point thus lies on the manifold $G_{c-1}(R^{N-1})$.

6. Let us examine the RG evolution of anomalous dimensionalities. On an RG trajectory, the anomalous dimensionalities of the fields Φ_{α} depend on the RG time τ. The matrix of anomalous dimensionalities is

$$\hat{\Delta}(\tau, \lambda) = t_{ab}(\tau)(\tau^{a}\tau^{b}).$$

In particular, for the models of class $\mathscr{C}^1$ we have

$$\Delta_A(\tau, \omega_J) = \frac{1}{2} g_{\mu\nu} \Big(\sum_{i \in A} R_i^{\mu}\Big)\Big(\sum_{j \in A} R_j^{\nu}\Big), \tag{14}$$

and for the spinor representations of the models of class $\mathscr{C}^2$ we have

$$16\Delta(\tau, \sigma) = c - 1 + \frac{1}{1+e^{\tau}}. \tag{15}$$

We have thus constructed a connected configuration space of AV models which is a simplified analog of the configuration space of a complete string theory. The remainder of the GLM program consists of constructing the $N \to \infty$ limit. The validity of that construction follows from the existence of a natural topology of an inductive limit, which may be the undivided configuration space. It is also necessary to include q-deformations and to make the generalization to an arbitrary Riemann surface Σ_g. By virtue of the local nature of the AV construction, there are no obstacles to this generalization.

We will take up a formulation of a dynamic principle on a AV configuration space in a separate paper.

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