Effect of motion of the optical medium in optical location

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(Submitted 3 February 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 6, 317–320 (25 March 1992)

A Fizeau experiment to demonstrate an "entrainment" of light by a moving medium in the optical location of prism reflectors on board space vehicles is discussed. The direction of the reflected light is derived analytically as a function of the velocity of the medium. The result differs from the standard formula for dealing with velocity aberration (the Bradley effect) by a coefficient associated with the refractive index of the medium filling the corner reflector. The theoretical conclusions have been tested and verified in an experiment involving a space vehicle.

In the optical location of space vehicles carrying prism retroreflectors, one is essentially carrying out a Fizeau experiment in a slightly altered form. The optical medium which is moving in this case (along with the space vehicle) with respect to the light source and the detector is a prism reflector. This reflector is usually a tetrahedron with three mutually orthogonal faces (a corner reflector). In this situation, the angle through which the reflected light is deflected becomes a function of the refractive index of the prism material. So far, we have not run across a description of this effect in the literature. Let us examine this effect in more detail and some of its practical consequences.

Figure 1 shows a tetrahedral reflector which is moving from right to left at a velocity v. We consider the reflection from this reflector of light which is propagating from bottom to top in plane P, which passes through edge AD and through the middle of the opposite face (B). Two parallel rays which go into the prism at points A and B propagate in opposite directions. From point A, the ray goes opposite the direction in which the reflector is moving, and at point B' it leaves the prism, having met the reflecting face coming toward this point. From B the ray goes along the direction in which the reflector is moving, and it leaves the reflector at A'.

Refraction occurs at the entrance face twice for each ray: as each ray enters the prism and as each ray leaves it. According to Snell's law for moving media,[1] the refraction angle is related to the velocity of the prism and to the refractive index of the material for light at the wavelength.

The refraction conditions are identical for the two rays involved entering the prism, so these refraction angles are equal. As the rays leave the prism, the angles at which they are incident on the interface between the two media depend on the propagation direction inside the prism.

Let us examine in this connection the reflection of rays from two faces: one real face and one equivalent face, which replaces an edge of the prism (this simplification allows us to go over from the three-dimensional problem to a two-dimensional problem).

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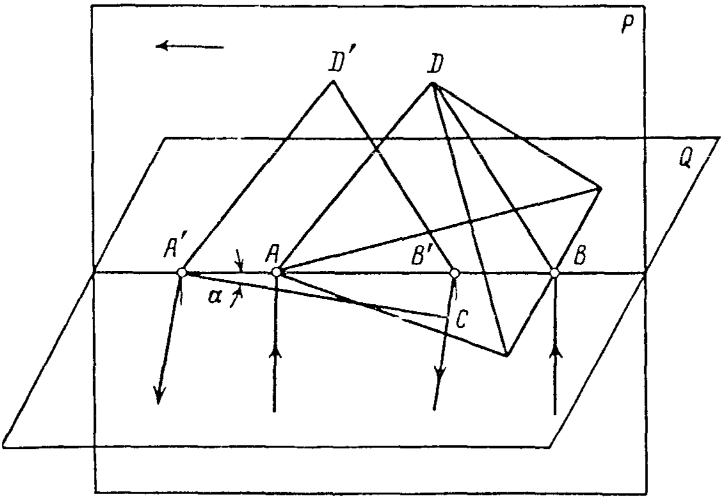

FIG. 1.

According to Refs. 2 and 3, the motion of the prism causes the reflection angle at one face to be greater than the angle of incidence, while it causes the reflection angle at the other face to be greater, by the same amount. As a result, after reflection from the two faces, the two rays will be propagating parallel to each other, along the original propagation direction.

The angles of incidence and refraction as the rays leave the prism will therefore be identical, regardless of the wavelength of the light.

We turn now to the time spent by the light propagating through the prism material. For rays orthogonal to the direction in which the prism is moving, there is no difference in velocity, so there is no difference in propagation time for a common wavelength. The situation is different for those rays which, after reflection from the lateral faces, are propagating along the entrance face of the prism.

We denote by Δt the propagation time in vacuum of light moving at a velocity C over a distance equal to the length of the entrance face of the prism, AB. We denote by Δt_1 and Δt_2 the propagation times of light inside the prism, with allowance for the velocities U_1 and U_2 in the prism material, over distances BA' and AB', respectively:

$$\Delta t_1 - \Delta t_2 = \frac{A'B}{U_1} - \frac{AB'}{U_2}.$$

Over the time which elapses while the ray which enters at B propagates to A, by the ray which enters at A and leaves at B' traverses a path length $B'C$ in vacuum. The propagation times from A to C through B and from B to A' are equal:

$$B'C = c(\Delta t_1 - \Delta t_2).$$

Line segments BA' and AB' are given by $BA' = AB + AA'$ and $AB' = AB - AA'$.

Their constituent parts in a linear measure can be written

$$AB = c\Delta t, \quad AA' = \left(\frac{v}{c}\right) AB = v\Delta t.$$

We then have $BA' = c\Delta t + v\Delta t$ and $AB' = c\Delta t - v\Delta t$. Using[4]

$$U_1 = \frac{\frac{c}{n} + v}{1 + \left(\frac{v}{c^2}\right)\frac{c}{n}} \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) \text{ and } U_2 \approx \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right),$$

we find

$$B'C = c\Delta t \left[\frac{c+v}{\frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)} - \frac{c-v}{\frac{c}{n} - v\left(1 - \frac{1}{n^2}\right)}\right].$$

The angles through which the outgoing wavefront is deflected from the incoming wavefront by the prism are given by

$$\sin\alpha = \frac{B'C}{A'B'} = \frac{B'C}{AB} = \frac{2vc(n - n^2 + 1)}{c^2 - \frac{v^2}{n^2}\left(1 - \frac{1}{n^2}\right)^2}.$$

For hollow corner reflectors ($n = 1$) this formula assumes the well-known form[5,6] for dealing with a velocity aberration (the Bradley effect):

$$\alpha = \frac{2v}{c}\rho.$$

For prism retroreflector devices we find, ignoring the small second term in the denominator with respect to the first,

$$\alpha = \frac{2v}{c}(n + 1 - n^2)\rho.$$

We have not seen velocity aberration dealt with in this manner elsewhere. It follows from this result that if a prism is made of a material with $n = 1.618$ then the reflected light should go back toward the light source, without a change in direction.

It is a complicated matter to test this dependence of the light deflection angle on n because isolated prisms are seldom placed on objects in space. When there is more than one prism—even just two—effects analogous to that observed in the case of a Michelson echelon grating arise: The reflection pattern is choppy, and the peak of the light distribution shifts away from its original direction when the angle of incidence of the rays on the prism is varied. Furthermore, if prisms with total internal reflection at their faces are used, the reflection pattern will again be choppy.[7] We would thus like to use isolated corner reflectors with aluminum-coated reflecting surfaces.

To test the validity of these theoretical conclusions, an experiment was carried out with the help of a space vehicle. Four corner reflectors were carried on board the vehicle, which was spatially stabilized along all three axes. Two of these reflectors were hollow, so only the Bradley effect should have occurred in their case (the refractive index was $n = 1$). The two others were made of quartz glass ($n = 1.45$). In the latter case the Bradley effect should partially cancel the Fizeau effect. The reflectors were installed in such a way that when light was directed at the space vehicle from the earth, a response signal could be obtained from only one reflector at a given instant. For example, when the vehicle was observed from the earth on one side of a plane passing through the vertical and oriented parallel to the orbital plane of the vehicle, the application of light resulted in the observation of, first, a signal reflected from a quartz reflector and then, after traversal of a crosspiece, reflection from a hollow reflector. When the vehicle was observed on the other side of this plane, the reflectors operated in the opposite order.

Since a plot of the change in the distance to the vehicle as a function of the time, during any passage, is symmetric with respect to the crosspiece, it is a simple matter to make a correct comparison of the intensities of the signals reflected from the hollow and quartz reflectors. The geometric dimensions of the reflectors and the precision with which they were fabricated were such that the width of the directional pattern of the reflected light, in the case of illumination by a laser with a wavelength of 532 nm, was about 4″ at the half-power level. The angular deviation caused by the Bradley effect, with allowance for the radial velocity of the vehicle and the aspect of the observer, was about 7″ on the average. The calculated difference angular deflection for the quartz reflector was about 2″ (the partial cancellation of the influence of the Bradley effect due to the Fizeau effect was taken into account). If this cancellation actually occurs, then the signal from the hollow reflector should be much weaker (by more than an order of magnitude) than the signal from the quartz reflector.

If the Fizeau effect has no influence, the reflected signals from the hollow and quartz reflectors should be identical, if the ranges are identical.

The experiments were carried out in June of 1989 (on 5, 10, and 14 June). More than 60 reflected signals were received. The results showed that, for identical average ranges and for identical observation conditions, the signals from the quartz reflector were more intense than those from the hollow reflector by an order of magnitude (in terms of the power), on the average. Moreover, the calculated absolute value of the received energy of the signals, with allowance for the effect of the atmosphere, differed from the observed value by a factor no greater than 2 or 3 in any observation run (the calculation for the quartz reflector was carried out under the assumption that there is a cancellation of the influence of the Bradley effect by the Fizeau effect). The theoretical predictions are thus supported by the results of actual experiments.

We draw this conclusion by the following arguments. Measurements of the patterns of the reflected signals on a special autocollimation television apparatus showed that the errors in the dihedral angles of all the reflectors are less than 0.2″ and could not cause any significant changes in the patterns. Tests of the prisms in their holders at various temperatures and various vacuum levels showed that the reflection pattern changes only slightly, even in the case of a thermal shock. These tests also showed that the space vehicle used in the experiment was in a solar-synchronous orbit and was stabilized. One pair of reflectors (one quartz reflector and one hollow reflector) was illuminated at all times, while the other pair was on the shadowed side of the vehicle. If the temperature had caused large changes in the patterns, the signals from the identical reflectors under different thermal conditions would have been different.

However, this is not what the experimental results revealed.

The most convincing piece of evidence is the very fact that the actual signal from the prism reflector was an order of magnitude stronger than (not weaker than) the actual and calculated values of the signal from the hollow reflector. If there was no cancellation of the Bradley effect by the Fizeau effect, this situation would have been possible only if the reflection pattern changed in such a way that (1) the pattern split into two lobes for each prism, (2) one of the lobes was deflected forward (in terms of the velocity of the vehicle), (3) the magnitude of the deflection of the lobes was equal to the magnitude of the deflection of the rays caused by the Bradley effect, and (4) the deflections of the lobes at the two prism reflectors were identical as the space vehicle passed over the locator. The probability for an exact coincidence is too small, and this possibility is refuted by the stability of the experimental results.

To assist in further experiments of this sort, we are prepared to furnish the necessary information to interested organizations which have location equipment.

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Translated by D. Parsons