Anisotropy of the fluctuation component of the penetration depth in exotic superconductors

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Fluctuation corrections to the magnetic-field penetration depth λ of exotic superconductors are derived. These corrections give rise to a temperature dependence of the relative anisotropy of λ near T_c. Experimental observation of such a temperature dependence would be evidence for an exotic superconductivity.

The exotic superconductors described by a multicomponent order parameter in the Ginzburg–Landau theory[1–3] should exhibit some distinctive anisotropic magnetic properties in many cases. Since these anisotropic properties would be qualitatively different from those of superconductors which can be described by the standard Ginzburg–Landau theory with an anisotropic mass tensor, a study of these properties could play an important role in identifying the nature of the superconducting pairing. Questions of this sort have recently attracted much interest in the literature in connection with discussions of the properties of certain heavy-fermion superconductors, Chevreul phases, and high-T_c superconductors. For example, a specific anisotropy of the upper critical field,[4–7] of the fluctuation diamagnetism,[7,8] and of the lower critical field[9,10] has been discussed.

Another interesting question is whether measurements of the anisotropy of the magnetic-field penetration depth λ would make it possible to distinguish a superconductor with a nontrivial type of pairing from an ordinary superconductor with an anisotropic mass tensor. Below we show that such a specific anisotropy of λ might occur in an exotic superconductor because of fluctuation corrections, even in cases in which there are no distinctive features in the anisotropy of λ if fluctuations are ignored.

A fluctuation component of the penetration depth in an anisotropic superconductor having a single-component complex order parameter was recently found by Buzdin and Vuyichit.[11] According to Ref. 11, the standard Ginzburg–Landau theory with an anisotropic mass tensor leads to the following result when slight fluctuations are taken into account:

$$\lambda_l^{-2} = \lambda_{o,l}^{-2}\left(1+\frac{1}{3}\sqrt{\frac{\mathrm{Gi}}{t}}\right). \tag{1}$$

Here $t=(T_c-T)/T_c$; $\mathrm{Gi}=2T_c m_1 m_2 m_3 b^2/(\pi^2\alpha)$ is the Ginzburg number; and $\lambda_{0,l}$ are the London depths when fluctuations are ignored (more precisely, the incorporation of fluctuations leads to no more than a slight renormalization of the constants in the expression for $\lambda_{0,l}^{-2}$).

The fluctuation component, with the characteristic temperature dependence, is represented by the second term in (1). An important point for the discussion below is that, according to expression (1), the incorporation of fluctuations does not lead to a change in the relative anisotropy of the penetration depths. Below we show that exotic superconductors differ from conventional superconductors in that the anisotropy of the fluctuation component of λ_i may be qualitatively different from the anisotropy of $\lambda_{0,i}$.

We consider a hexagonal superconductor with strong spin-orbit coupling and a nontrivial pairing. The order parameter for such a superconductor has two complex components (η_1,η_2). The Ginzburg–Landau functional is written

$$H[\eta]=\int dV[-a\eta_i\eta_i^* + \beta_1(\eta_i\eta_i^*)^2+\beta_2|\eta_i\eta_i|^2+K_1p_i^*\eta_j^*p_i\eta_j$$
$$+K_2p_i^*\eta_i^*p_j\eta_j+K_3p_i^*\eta_j^*p_j\eta_i+K_4p_z^*\eta_i^*p_z\eta_i], \tag{2}$$

where $p_1=p_x$; $p_2=p_y$; $\vec{p}=-i\nabla-(2e/c)\vec{A}$; $a=\alpha t$; $\alpha>0$;

$$\beta_1>0;\quad \beta_1+\beta_2>0;\quad K_1+K_2+K_3>|K_2|;\quad K_1>|K_3|;\quad K_4>0. \tag{3}$$

If $\beta_2>0$, a minimum of the free energy in (2) in the absence of a magnetic field corresponds to a homogeneous superconducting phase, $(\eta_1,\eta_2)\propto(1,\pm i)$. In the case $\beta_2<0$ in contrast, we have a state $(\eta_1,\eta_2)\propto(1.0)$

In general, the expression for the current density which follows from (2) is comparatively complex. In the London approximation, which is sufficient for the discussion below (we assume $\kappa\gg1$ everywhere, where κ is the Ginzburg–Landau parameter), the current density is given by the simple expression

$$j_l=\frac{e}{2m}n_{li}^s\left(\nabla_i\Phi-\frac{2e}{c}A_i\right). \tag{4}$$

Here the tensor n_{li}^s is the density of superconducting electrons.[12] The eigenvalues n_l^s of this tensor are related in a simple way to the corresponding penetration depths:

$$n_l^s = \frac{mc^2}{4\pi e^2}\lambda_{o,l}^{-2}, \quad l = x, y, z. \tag{5}$$

The particular crystal faces and magnetic field orientations to which the quantities $\lambda_{0,l}$ correspond can easily be found from (4) and (5) in the coordinate system in which the tensor $n_{l,i}^s$ is diagonal.

In the case $\beta_2 > 0$ we find from (2), (4), and (5)

$$\lambda_{o,l}^{-2} = \frac{16\pi e^2 a}{\beta_1 c^2}\gamma_l, \quad \vec{\gamma} = (K_1(1+C), K_1(1+C), K_4), \tag{6}$$

where $C = (K_2 + K_3)/(2K_1)$. The anisotropy of the penetration depths described by expression (6) is of the same nature as in an ordinary uniaxial superconductor. In the latter, two of the three eigenvalues of the mass tensor (and thus the penetration depths) are equal.

If $\beta_2 < 0$, we find, by analogy with (6),

$$\lambda_{0,l}^{-2} = \frac{16\pi e^2 a}{(\beta_1 + \beta_2)c^2}\zeta_l, \quad \vec{\zeta} = (K_{123}, K_1, K_4), \tag{7}$$

where $K_{123} = K_1 + K_2 + K_3$. The anisotropy of the penetration depth described by expression (7) reflects a disrupted hexagonal symmetry of the (1,0) superconducting state. Here all three penetration depths are different. This circumstance is a specific feature of the anisotropy of the depth to which a magnetic field penetrates into a hexagonal exotic superconductor whose unperturbed state is the (1,0) state.

We turn now to the fluctuation component of the penetration depth. When fluctuations of the order parameter are taken into account, the free energy is given by

$$F = -T \ln \int e^{-H[\eta]/T} D\eta. \tag{8}$$

The effective Hamiltonian $H[\eta]$ here is defined in (2).

Further calculations are carried out in the Gaussian approximation. In many regards, these further calculations are like those which were carried out in Ref. 11 for the case of ordinary superconductors with an anisotropic mass tensor. Since the integral in (8) is dominated by length scales $\sim\xi$ (ξ is the coherence length), we can use the approximation $\vec{A} = \text{const}$ in evaluating functional integral (8). In a coordinate system in which the tensor n_{li}^s is diagonal, we can thus use the following simple expression to calculate the penetration depths λ_i:

$$\lambda_l^{-2} = \frac{4\pi}{V} \cdot \frac{\partial^2 F}{\partial A_l^2}. \tag{9}$$

We first consider the case $\beta_2 < 0$.

The quantity $(K_2 - K_3)/K_1$ is known to be extremely small near the Fermi surface, because of the approximate particle-hole symmetry. At this point we set $K_2 = K_3$.

It then follows from (3) that the quantity $C = (K_2 + K_3)/(2K_1) = K_2/K_1$ is bounded by the inequalities $-1/3 < C < 1$. We calculate the free energy in (8) in the Gaussian approximation for the fluctuations. We use a power series in the parameter C. Retaining terms of first order in C, we find the following expression for $\lambda_l^{-2}(T)$:

$$\lambda_l^{-2}(T) = \lambda_{0,l}^{-2}\left[1 + \frac{1}{3}\sqrt{\frac{\mathrm{Gi}}{t}} f_l(C, B)\right], \tag{10}$$

where

$$f_{x,y} = 1 - C\left(1 \pm \frac{\sqrt{B}}{1+\sqrt{B}}\right); \quad f_z = 1 - C$$

$$\mathrm{Gi} = \frac{T_c(\beta_1 + \beta_2)^2}{8\pi^2 K_1^2 K_4 \alpha}(1 + \sqrt{B})^2; \quad B = \frac{|\beta_2|}{\beta_1 - |\beta_2|}. \tag{11}$$

The quantity $\lambda_{0,l}^{-2}(T)$ is given in (7).

Comparison of (1) and (10) shows that fluctuations in the order parameter in exotic superconductors give rise to a temperature dependence of the relative anisotropy of the penetration depths, i.e., to a temperature dependence of quantities of the type $\lambda_{x,y}(T)/\lambda_z(T)$. In this regard, exotic superconductors stand in contrast with conventional anisotropic superconductors.

In the case $\beta_2 > 0$, a power series in the parameter $\epsilon = C/(1 + C)$ arises in a natural way in the calculation of the fluctuation component of the penetration depth. Under the condition $K_2 = K_3$, we find $|\epsilon| < 1/2$ from (3). Taking small terms of second order in ϵ into account, we find

$$\lambda_l^{-2} = \lambda_{0,l}^{-2}\left[1 + \frac{1}{3}\sqrt{\frac{\mathrm{Gi}}{t}} g_l(\epsilon, b)\right], \tag{12}$$

where

$$g_x = g_y = 1 - \epsilon - 24\epsilon^2 P_1(b); \quad g_z = 1 - \epsilon - 24\epsilon^2 P_2(b),$$

$$\mathrm{Gi} = T_c\beta_1^2/(8\pi^2 K_1^2 K_4 \alpha); \quad b = \beta_2/\beta_1, \tag{13}$$

$$P_1(b) = -\frac{168b^2 + 504b^{3/2} + 672b + 497\sqrt{b} + 167}{210(1+\sqrt{b})^3}, \tag{14}$$

$$P_2(b) = \frac{56b^2 + 168b^{3/2} + 224b + 161\sqrt{b} + 47}{210(1+\sqrt{b})^3}. \tag{15}$$

The temperature dependence of the relative anisotropy of the penetration depths, $\lambda_{x,y}/\lambda_z$, arises here only when we retain terms of second order in the small parameter ϵ. According to (12) and (13), fluctuations do not disrupt the original equality $\lambda_x = \lambda_y$, which prevails in the case $\beta_2 > 0$ [see (6)].

According to the results found above, a temperature dependence of the relative anisotropy of the penetration depths near T_c would be evidence of an exotic superconductivity. To the best of our knowledge, there has been no study of the fluctuation component of the penetration depth. Experimentally, a weak temperature dependence of the relative anisotropy would be easier to detect than a specific temperature-dependent correction to the absolute values of the penetration depths. One can thus hope that experiments of this type could be carried out in order to identify exotic superconductors.

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