Dynamics of the magnetization reversal of a cylinder in an alternating magnetic field

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The magnetization reversal of a ferromagnetic cylinder in an auxiliary alternating magnetic field is studied. Self-excited oscillations of the magnetization arise in a certain interval of values of the static field. The field at which the magnetization reverses is determined by an instability of these self-excited oscillations.

The first nonlinear calculation on the magnetization reversal of a cylinder using a model of uniform rotation was carried out by Stoner and Wohlfarth.[1] They showed that the magnetization undergoes an irreversible 180° rotation at a certain critical field $H_{SW} = -[2K/M_s + (N_p - N_z)M_s]$, ($K$ is the uniaxial-anisotropy constant, M_s is the saturation magnetization, and N_z and N_p are the longitudinal and transverse demagnetizing factors; this critical field is directed parallel to the axis of the cylinder). In the present letter we report a study of the magnetization reversal of a cylinder in a circularly polarized auxiliary field which is applied in the plane perpendicular to the axis of the cylinder.

1. In spherical coordinates (θ is the polar angle, reckoned from the z axis, and ϕ is the azimuthal angle), and in the uniform rotation model, the energy E is the sum of three components: the uniaxial-anisotropy energy, the magnetostatic energy, and the energy of the magnetic material in the field. We thus have

$$E=\int dV\{K\sin^2\theta+\frac{1}{2}N_zM_s^2\cos^2\theta+\frac{1}{2}N_pM_s^2\sin^2\theta$$
$$-H_zM_s\cos\theta-h_0M_s\sin\theta\cos(\phi-\omega t)\},$$

where H_z is the static magnetic field, which is directed along the axis of the cylinder, and h_0 and ω are respectively the amplitude and frequency of the alternating field. The Landau–Lifshitz equations, written in dimensionless form in a coordinate system which is rotating along with the high-frequency field [for this purpose we set $\phi(t)=\omega t+\psi(t)$], are

$$-\dot{\theta}-\alpha\dot{\psi}\sin\theta=\alpha\omega\sin\theta+h_0\sin\psi,$$

$$\dot{\psi}\sin\theta-\alpha\dot{\theta}=\sin\theta\cos\theta+(H_z-\omega)\sin\theta-h_0\cos\theta\cos\psi, \tag{1}$$

where H_z and h_0 are normalized to $|H_{SW}|$, ω is normalized to $\gamma|H_{SW}|$ (γ is the gyroratio), and α is a dimensionless damping constant. When the alternating field is circularly polarized in the plane perpendicular to the easy axis of the cylinder, the nonautonomous initial system of equations can be reduced, through a transformation to a rotating coordinate system, to the autonomous system of equations in (1). The further analysis is substantially simplified as a result. Here ψ is the phase lag of the magnetization precession behind the high-frequency field.

2. We have carried out a numerical study of the dynamics of the magnetization reversal as described by system of equations (1). Figure 1 shows the polar angle (θ) of the magnetization precession cone versus the constant magnetic field, which is lowered in a quasistatic fashion. This constant field is initially directed along the positive z direction. The amplitude and frequency of the alternating field are fixed and are given in dimensionless units in Fig. 1. At large and positive values of H_z, the magnetization precesses at the frequency of the external field, with a precession angle which is constant (at constant H_z) and small. The progressive lowering of the magnetizing field results in an increase in the steady-state precession angle, and at a certain critical value of H_z this solution becomes unstable. However, instead of the magnetization rotation which we would expect on the basis of the simple linear theory,[2] self-excited oscillations arise in the system, in a hard fashion. The meaning here is that slow oscillations of the angle of the precession cone are superimposed on the fast precession of the

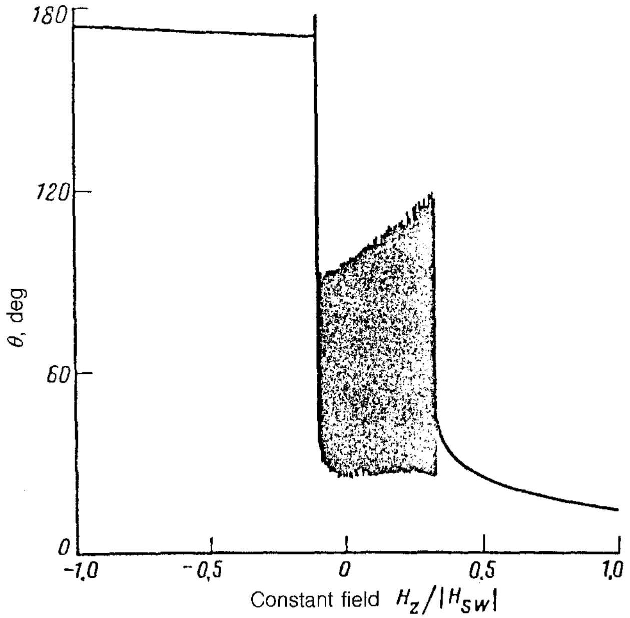

FIG. 1. The polar angle (θ) of the magnetization-precession cone versus the strength of the "constant" magnetic field, which is being reduced in a quasistatic fashion as time elapses. This constant field is directed along the axis of the cylinder. The field is normalized to the value of $|H_{SW}|$. The values of the amplitude and frequency of the additional alternating field are fixed (at $h_0 = 0.3$ and $\omega = 0.8$, in dimensionless units). The damping constant is $\alpha = 0.2$.

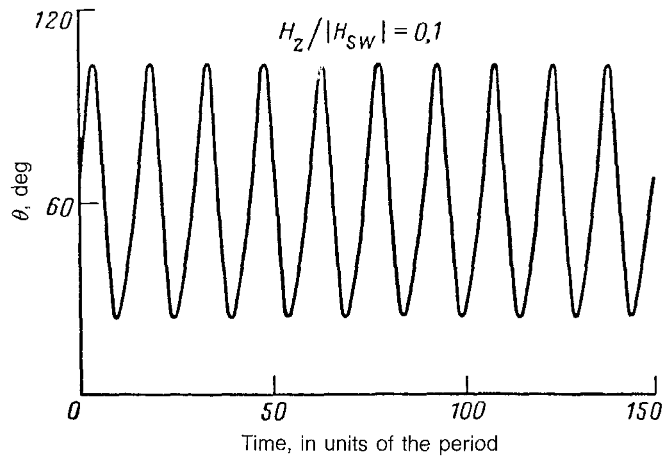

FIG. 2. Periodic (in time) oscillations of the angle of the precession cone at a constant value of H_z ($\omega = 0.8$, $h_0 = 0.3$, $\alpha = 0.2$). The frequency of the self-excited oscillations is much lower than the frequency of the alternating field.

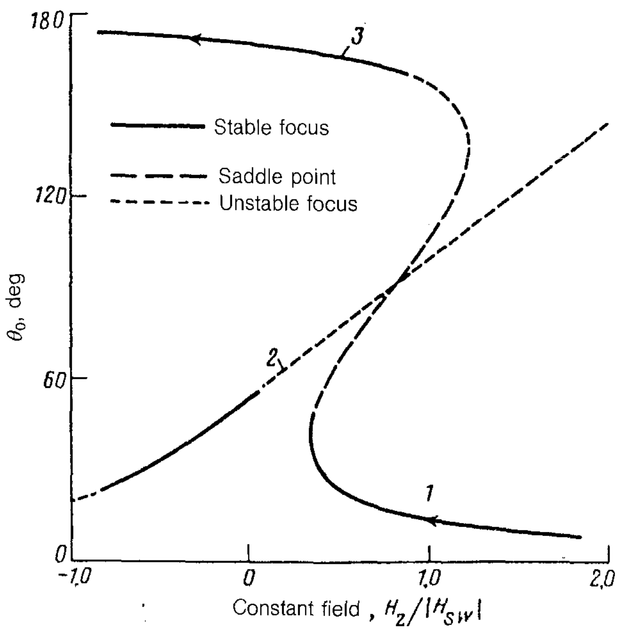

FIG. 3. Steady-state solutions θ_0 of Eqs. (1) versus the strength of the constant magnetic field ($\omega = 0.8$, $h_0 = 0.3$, $\alpha = 0.2$).

magnetization, at the frequency of the external field (the oscillations of the angle of the precession cone are slow in comparison with ω). We wish to stress that, if the sweep of the magnetic field is stopped at a certain value corresponding to the region with self-excited oscillations, these oscillations will persist, with a constant amplitude and with a frequency well below the frequency of the external field (Fig. 2). At a certain value of the constant field, the regime of self-excited oscillations becomes unstable, and the magnetization reverses rapidly.

3. To give this magnetization-reversal scenario a theoretical basis, we examine the stability of steady-state solutions of Eqs. (1). Such solutions correspond to a precession with a constant angle θ_0. Figure 3 shows θ_0 as a function of the strength of the constant magnetic field at fixed values of ω and h_0 (the same values as in Fig. 1). We see that $\theta_0(H_z)$ is a multivalued function. Also shown here are various types of equilibrium positions which are realized in this system. Their stability is determined in the standard way, by calculating the eigenvalues of a linearization matrix.[3] As the field is lowered, the imaging point evidently moves clockwise along lower branch (1) of the θ_0 (H_z) curve, until this solution disappears as the result of a bifurcation consisting of the coalescence of a saddle point and a node. The imaging point should then move to the upper branch (3), since the intermediate equilibrium position (2) is an unstable focus. This conclusion obviously contradicts the results of the numerical calculations (Fig. 1), according to which self-excited oscillations arise in the system at this point. The meaning here is that in the phase space of the magnetic material there exists a stable limiting cycle around unstable intermediate focus 2. This cycle encloses the magnetization. An analysis of the reason for the appearance of this cycle goes beyond the scope of the present letter and will be reported in a later paper. The subsequent evolution of the magnetization is determined by the stability of the limiting cycle. For the given values of h_0 and ω, a decrease in H_z leads to a strict loss of stability (Fig. 1), with the result that the magnetization reverses.

In summary, when a single-domain ferromagnetic particle undergoes magnetization reversal in the presence of an auxiliary alternating field, self-excited oscillations arise in a certain interval of values of the constant field. The field at which the magnetization reverses is not the same as the Stoner–Wohlfarth critical field. For given values of h_0 and ω, the magnetization-reversal field is much lower (in absolute value) than $|H_{SW}|$. The effect observed here might turn out to be useful for controlling the coercive force of single-domain particles used in magnetic recording.

[1]E. C. Stoner and E. P. Wohlfarth, Philos. Trans. Roy. Soc. London **240**, 599 (1948).
[2]W. F. Brown, Jr., *Micromagnetics*, Wiley Interscience, New York, 1963.
[3]V. I. Arnol'd, *Additional Topics in the Theory of Ordinary Differential Equations*, Nauka, Moscow, 1978.

Translated by D. Parsons